Bazinga! Physics

Round 1: Prelims

1. A small sphere falls from rest in a viscous fluid. Due to friction, heat is produced as the sphere moves relative to the fluid. The rate of heat production at the terminal velocity is directly proportional to the *n*^th power of the sphere's radius. Find n.

   Answer: 5

   Solution:

   Terminal velocity: $v_t = \frac{2r^2 g(\rho_s - \rho_L)}{9\eta}$

   Viscous force: $F = 6\pi\eta r v_t$

   Rate of heat production (power) = $F v_t \propto r^5$

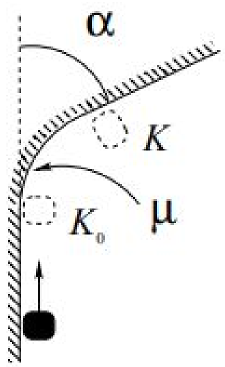
2. A comet (assumed to be in an elliptical orbit around the sun) is at a distance of 0.4 AU from the sun at the perihelion. If the time period of the comet is 125 years, what is the aphelion distance? AU: Distance between earth and sun. (Perihelion: Shortest distance between the comet and the sun, Aphelion: The largest distance between the comet and sun.)

   Solution: Let Aphelion distance be y AU. define r=(0.4+y)/2. Now using Kepler's law T^2 is proportional to R^3, and the fact that 1AU is the distance between the earth and the sun where T is 365 days, we get y=49.6 AU.

3. A ball is thrown straight up and caught at the same height at which it was released. Air resistance is a function of velocity. Did the ball spend more time during its flight going up or going down? Justify.

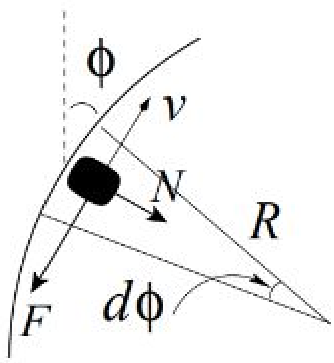
   Solution: Due to drag force, the energy of the ball is continuosly decreasing. Hence at each point in the path, the energy while coming down is less than the energy of the ball while going up at the same position. Hence Accordingly, velocity will be lesser while coming down and hence time of descent will take more time.

4. An object is sliding on the floor along a wall. The floor is frictionless, but there is friction between the object and the wall, and the friction coefficient is μ. The wall makes a smooth turn, by angle α. Just before the turn, the kinetic energy of the object is K_0. Find the kinetic energy right after the turn is completed.



Solution: (Ice Hockey Puck Curve)
Consider a point where the velocity of the object makes angle φ with the original direction, and take the curvature radius here to be R. Relevant equations are:

$$m\frac{v^2}{R} = N \qquad and \qquad F = \mu N$$



and thus

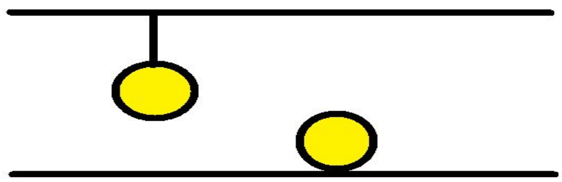
$$d\left(\frac{mv^2}{2}\right) = -FRd\phi = -\mu m v^2 d\phi$$

giving

$$\boxed{K = K_0 e^{-2\mu\alpha}}$$

This answer does not depend on the exact details of the curve, its shape or length, as long as there are no sharp corners anywhere. The only parameter that matters is the angle between the final and initial velocities!

5. Consider two identical homogeneous metal balls, A and B, with the same initial temperatures. One of them is at rest on a horizontal plane, while the second one hangs on a thread. The same quantities of heat have been supplied to both balls. Are the final temperatures of the balls the same or not? Justify your answer. (All kinds of heat losses are absent.)



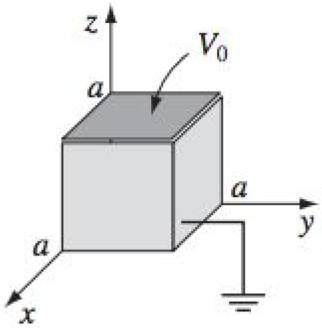
Solution: No, the temperature of the ball hanging from the thread is more as it experiences a decrease in it's potential energy while the one on the floor experiences an increase(this is because heating makes the spheres expand which raises the center of mass of sphere kept on ground while lowers the same for the sphere hanging from the wall. But since the total energy must be conserved and both balls start with the same initial temperatures,hence the answer follows.

6. A drop of water is in a vacuum. You may assume the desired variables but please mention what they stand for.

a) What is the pressure difference between its top and bottom points assuming the drop falls freely under gravity?
b) What is the pressure difference between its top and bottom points if it is falling under the influence of gravity at a constant velocity.

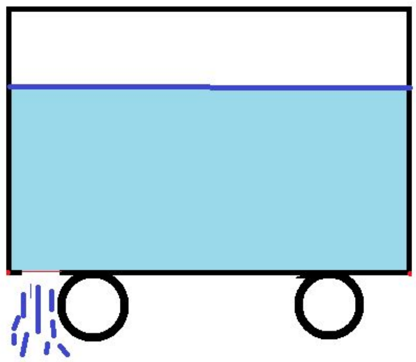
Solution: 0 and pgh, where p is the density of the bubble and h is the height of the water droplet.
Explanation: When the drop is free falling, There is no relative acceleration between the two ends and hence pressure difference will be 0.
In the second case, one can imagine the drop kept in a box, the pressure difference between the top and bottom ends will be pgh. Since all inertial frames are equivalent, hence the result shouldn't change even if the box is moved at constant velocity. Here, the role of a box is performed by the drag force.

7. A cubical box consists of 5 metal faces which are welded together and grounded. The top, made from a sheet of metal, is kept at a potential V_o. What is the potential at the center of the box? Justify.



Solution: $\frac{V_0}{6}$. Consider the superposition of 6 such cubes, one with V_0 on each face of the cube. Resulting cube has V_0 on all it's faces and hence potential at the centre is V_0 too. Evidently, the potential at centre originally is $\frac{V_0}{6}$.

8. Consider a truck full of water at rest. It has a hole at the bottom on the left side as shown in the figure. In which direction will the truck move just after the water starts falling out of the hole. Justify your answer.



Solution: Just as the water starts to fall, the water on the right rushes to the left to fill the void.
Hence, the center of mass of the water in the bucket moves to the left. But since

there's no external force, hence center of mass of the system must be the same(note that gravity is an external force but is balanced by the normal force by the floor of the truck and the floor on which the truck is resting).Hence the truck moves to the left.

9. Identical ordinary ice cubes are put into two quite large beakers, one of which contains tap water, the other a strong brine solution. The liquids have the same volume, and both of them are at room temperature. In which beaker will the ice cube melt more quickly and why?

Solution: The ice cubes float on the surface of the liquid in both vessels. In the first beaker, the density of the melted water from the ice cube, at a temperature of 0 C, is greater than that of the tap water at room temperature, and so it sinks to the bottom of the vessel and is replaced by room-temperature water. The convection currents arising in this way assist the melting. In the second beaker, the density of the brine is greater than that of the cold meltwater, and so the latter does not sink but remains around the ice cube. In this case, convection does not set in, and the melting of the ice cube is much slower than it was in the tap water.

10. Consider a sprinkler. It's common to you that when water rushes out, the sprinkler rotates in the opposite direction. Now consider a sprinkler immersed in an ocean. It sucks in water, contrary to the conventional ones. In which direction will the sprinkler rotate? Justify.

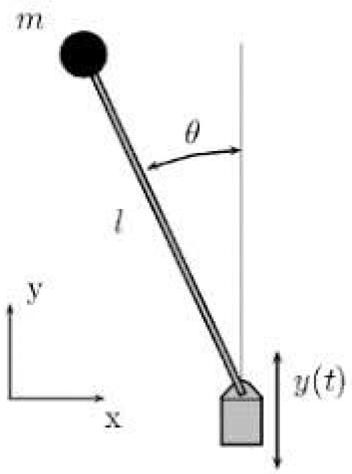
Solution: The Feynman Sprinkler! Apart from an initial jiggle about its mean point, the sprinkler won't move. This famous question drew the attention of many greats including Feynman, hence the name.
The reason for the initial wobble is that there will be a pressure difference that will be driving the liquid in the sprinkler. But that same pressure difference will also produce a net force on the sprinkler because one of its end is open. This will produce the initial wobble, but as liquid rushes in, it is balanced by the force of the incoming liquid. One can check that.

Round 2: Brief Thought Round:

1. Consider a pendulum as shown below whose pivot is performing SHM in the vertical direction.

   Surprisingly, the pendulum is stable in the vertically upright Position! Why?

   Is the bottom point also at stable equilibrium? Present your arguments.



Solution:

We can think of the pendulum motion as broken up into two types: slow and fast. The slow motion is the large left-right rotations that a pendulum with a stationary pivot would exhibit, and the fast motion that is due to the rapidly oscillating pivot. To go along with these two types of motion we will have to consider two different time scales for the pendulum motion. Imagine a pendulum that is deflected from the vertical by an angle Θ on average. Instantaneously, this deflection angle can be broken into the two parts, with each part created by one of the types of motion just discussed. There will be the slow deflection angle $\theta(t)$ caused by the left-right rotation, and the small, quickly changing angle $\delta(t)$ created by the fast pivot oscillations. See Fig. 1.2(b 1&2). We can now write the instantaneous angle as $\Theta(t) = \theta(t) + \delta(t)$. Note that over one period of the fast oscillation, $<\delta(t)>= 0$, and the slow motion remains essentially constant, so $<\theta(t)>= \Theta$. We can see from Fig. 1.2(b1) that we can use the law of sines and the fact that δ is small to write $\delta(t)$ geometrically as:

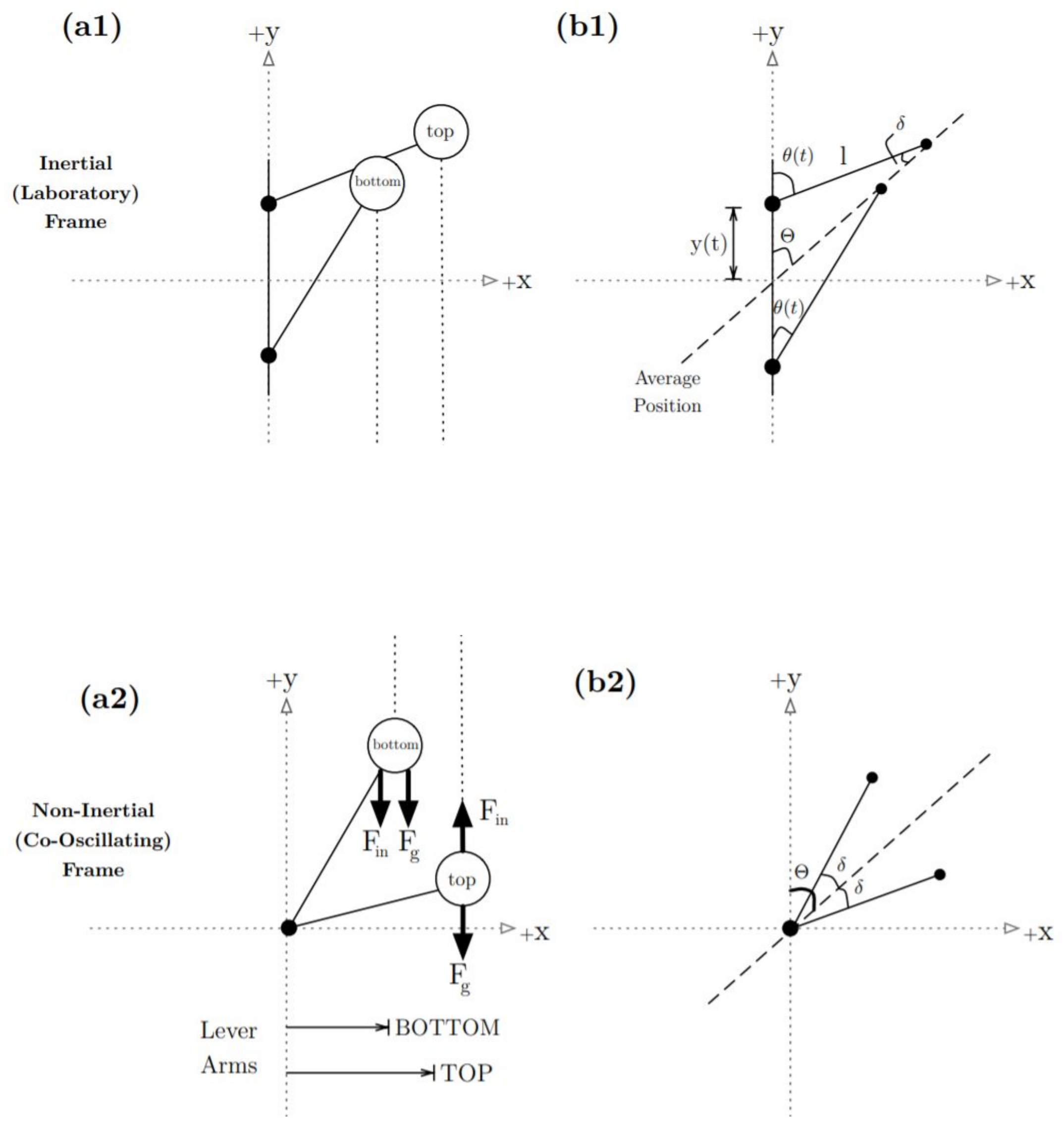
$$\frac{y(t)}{\sin\delta} = \frac{l}{\sin\Theta} \implies \delta = \frac{y(t)}{l}\sin\Theta \implies \delta = \frac{A}{l}\sin(\Theta)\sin(\omega t), \tag{1.1}$$

where $y(t) = A\sin(\omega t)$ is the motion of the pivot and l is the length to the center of mass of the pendulum. We can then write the instantaneous deflection angle as:

$$\Theta(t) = \theta(t) + \delta(t) = \theta(t) - \frac{A}{l}\sin(\theta)\sin(\omega t). \tag{1.2}$$

Since δ is small, we can say that

$$\sin(\Theta) = \sin(\theta+\delta) = \sin(\theta)\cos(\delta) + \sin(\delta)\cos(\theta) = \sin(\theta) + \delta\cos(\theta). \tag{1.3}$$



As you can see in a2, the net force due to F_{in} is 0. But the torque due to F_{in} is non-zero and is in upward direction as can be seen from the picture. For every F_{in} acting on the body in upward direction there exists another force with same magnitude acting in the downward direction. But the force in downward direction is closer to the origin compared to the force in upward direction, which causes a net torque in the upward direction. When this torque becomes large enough it balances the torque due to gravity and the upright position becomes stable. If you do exact calculations you will get that following relation must be satisfied for the pendulum to be stable-

$$A\omega \;>\; \sqrt{2gl}.$$

Also, doing similar analysis one sees that the bottom position remains a stable equilibrium position.

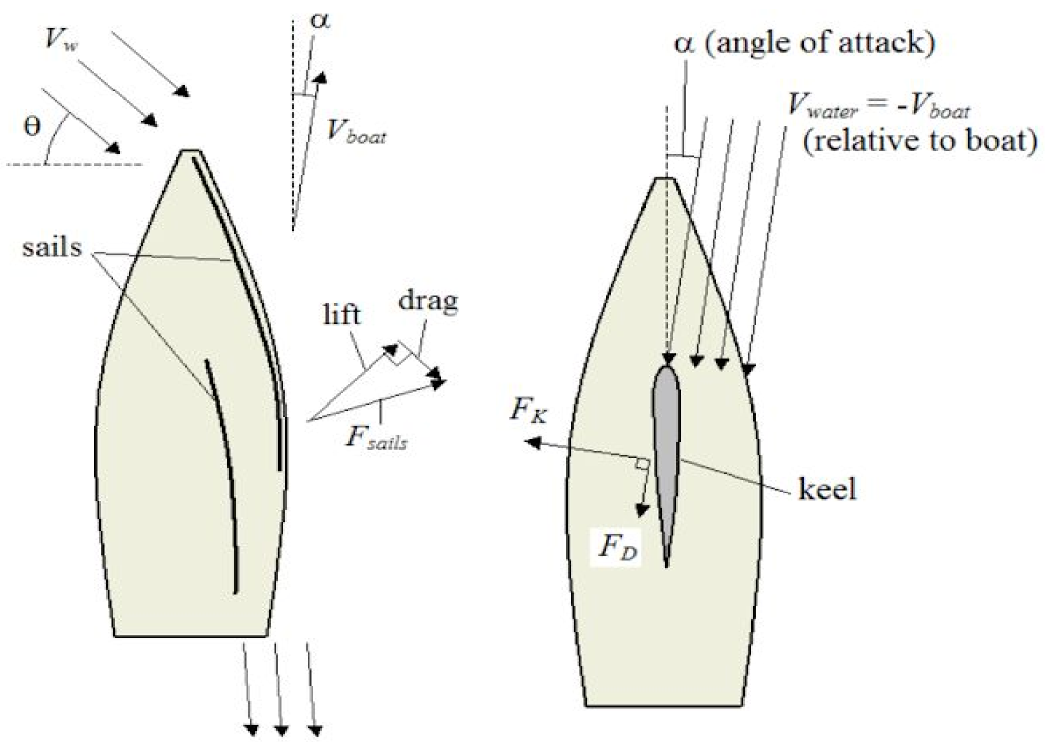
2. Can a ship using sails move effectively opposite to the wind direction?Justify your answer.

Can the ship move faster than the wind it is using to propel itself? What angle would you keep between the ship's and wind's direction to achieve most efficient results?

Can you suggest a more direct method to travel opposite to the wind direction?(please abstain from using an engine:P)

Solution: The figure below shows the general case where the wind V_w is blowing at an angle θ from the horizontal. This creates a resultant force on the sails, denoted by F_{sails}, which points in the direction shown.

Note that V_w is the wind velocity relative to the boat. This is not to be confused with the wind velocity relative to the water. If the boat is moving, these velocities are not the same.



The force F_{sails} is broken down into two components: *lift* (which acts perpendicular to the wind direction V_w) and *drag* (which acts parallel to the wind direction V_w). Lift and drag are defined as acting in these two directions; a convention commonly used in the literature with regards to air flow over a wing. As it turns out, air flow over a sail is very similar to air flow over a wing. The two sails (as shown) are oriented so as to optimize the air flow around and between the sails and generate as much "push" force as possible, to move the sailboat forward. The flexibility of the sails allows them to mimic the behaviour of a wing and be oriented in a variety of different positions to get the most "push", depending on which direction the wind is blowing.

The velocity of the sailboat (relative to the water) is denoted by V_{boat}. This velocity is in a direction skewed slightly to the right of the center line of the boat, by an angle α. This means that the sailboat does not travel "head on" through the water. It is necessary that the sailboat travel slightly off center (with some sideways movement) because it enables the keel to generate the necessary counter-force to resist the sideways force exerted on the sails by the wind. Consequently, some sideways movement is inevitable, but the keel keeps it (and hence α) as small as possible.

The keel behaves like an underwater wing and the same basic physics applies, as explained above for the sail. The force F_K is the force exerted by the water on the keel and hull, due to the angle of attack α the keel makes with the water streamlines. Most of this force is (intentionally) due to the keel, which is large and made to resemble a wing to create as much counter-force as possible in order to minimize sideways movement of the boat. Using the same convention as before, F_K is defined as perpendicular to the direction of flow of water V_{water}.

The force F_D is the drag force exerted by the water on the keel and hull, due to the angle α the keel makes with the water streamlines. Using the same convention as before, F_D is defined as parallel to the direction of flow of water V_{water}.

We can now sum all the forces acting on the sailboat.The sideways motion of the ship is counteracted by F_K. While the forward component of F_{sails} propels the ship forward and in steady state is balanced by backward force of keel.

It should be noted that it's impossible for a sailboat to travel directly into the wind (θ = 90° in the above figure). This is because the resultant force F_{sails} has no forward component. Instead, it has a backward component meaning the sailboat would travel *backwards*. But you could effectively travel forward in a zigzag manner. So there is an upper limit on how large θ can be. For very efficient sailboats this upper limit is around 60°.

The velocity of the wind relative to the boat (V_w) depends on the speed of the boat (V_{boat}). One can find V_w using vector addition. If one knows V_{boat} and the wind velocity relative to the water (call this V_{w1}), we can use the following vector formula to calculate V_w:

$V_w = V_{w1} - V_{boat}$

The optimal wind angle for greatest sailboat speed is when V_w is blowing from the side (θ = 0). There are two main reasons for this. The first reason is because the lift force is pointing in the forward direction (parallel to the boat center line). The second reason is because the forward push force (forward component of F_{sails}) remains fairly constant as V_{boat} increases. This means that the sailboat can accelerate to a high speed, since the forward push force is largely independent of sailboat speed V_{boat}.

But if the wind is blowing from behind the boat, V_w (and therefore wind force) is very dependent on V_{boat}. The faster the boat moves forward, the lower the relative wind velocity V_w and the lower the wind force. Therefore, the boats speed can never exceed wind speed. In fact, the top speed will be significantly less than this.

However, if the wind is blowing from the side it is actually possible for V_{boat} to be greater than V_w (in magnitude). This is because the push force is great enough and constant enough to propel the sailboat to a high speed.

A method to travel in the opposite direction of wind would be to use a frustum whose bigger hole is directed towards the wind while the smaller end is directed away from it. At first glance it may seem like it would push it away but if you assume an ideal fluid using the conservation of mass it can be shown that the ship will experience a feeble force in the opposite direction of the wind flow.

3. A cartwheel of radius 50 m has 12 spokes, assumed to be of negligible width. It rolls along level ground without slipping, and the speed of its axle is 15 m s−1.

Hawkeye wants to fire an arrow(20 cm long) such that it passes between the spokes of the wheel unimpeded.

Use a **graphical approach** to estimate the minimum speed he must fire it at.Neglect any vertical displacement of the bolt.

Solution: The problem is equivalent to the following question. For how long may we put one of our fingers, held stationary with respect to the ground, between the spokes of the rolling wheel without touching them?

The motion of the spokes of the rolling cartwheel is quite complicated, and an analytic solution to the problem is difficult. Instead of attempting this, let us tackle the question using a graphical method. To do this, we stick a paper disc onto the base of a convenient circular cylinder (e.g. a jar or a tin), and, while the cylinder is rolled along a table, the tip of a felt pen (stationary with respect to the table) is held against the paper disc, as shown in Fig 1.

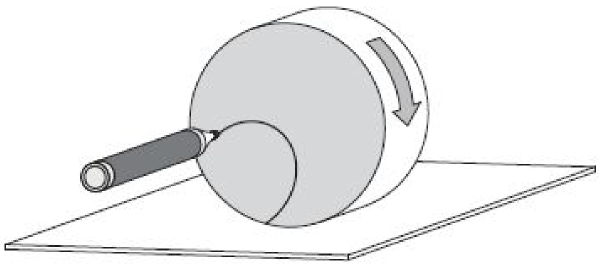


Fig 1.

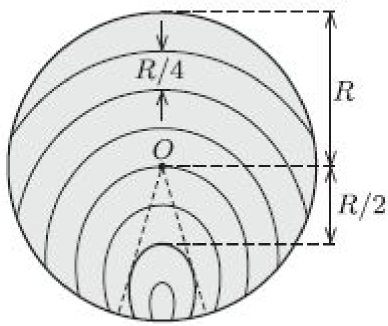


Fig 2.

What we will get are the curves shown in Fig 2. The different curves corresponding to different heights of the pen tip above the surface of the table. In the actual figure shown, the height of the pen has been changed between 0 and 2R in eight steps of R/4, where R is the radius of the rolling cylinder.

In order to maximise the time available for the passage of the bolt, we have to choose the curve for which the horizontal displacement of the rolling cylinder is maximal, but subject to the constraint that the pen tip remains between two successive ‘spokes’ (on the paper disc, the longest curve that lies entirely between two radii with an angle of 360/12 = 30◦ between them). We may choose only from those curves that do not cross any spokes, or, more precisely, if they do, then only its length between the crossing points may be considered. The required optimal curve can be identified as the one that is just touched by the two successive ‘spokes’. As seen in Fig. 2, this is a curve very close to the one for which the pen height was R/2. From this heuristic approach, we can conclude that the optimal result is obtained if the pen tip is at a height of approximately 0.5R above the level of the table.

Thus, the crossbow bolt should be fired between two spokes at a distance of 0.5R from the axis of the wheel. At this height, the length of a horizontal chord of the wheel is $2R\sqrt{(1-0.5^2)} \approx 1.7R$; the bolt may remain between the spokes for as long as it takes the axis to move this distance. The time for this is

$$t = \frac{1.7 \times 0.5 \text{ m}}{15 \text{ m s}^{-1}} \approx 0.057 \text{ s}.$$

So the speed of the bolt should be at least

$$v = \frac{0.2\ \mathrm{m}}{0.057\ \mathrm{s}} \approx 3.5\ \mathrm{m\,s^{-1}}.$$

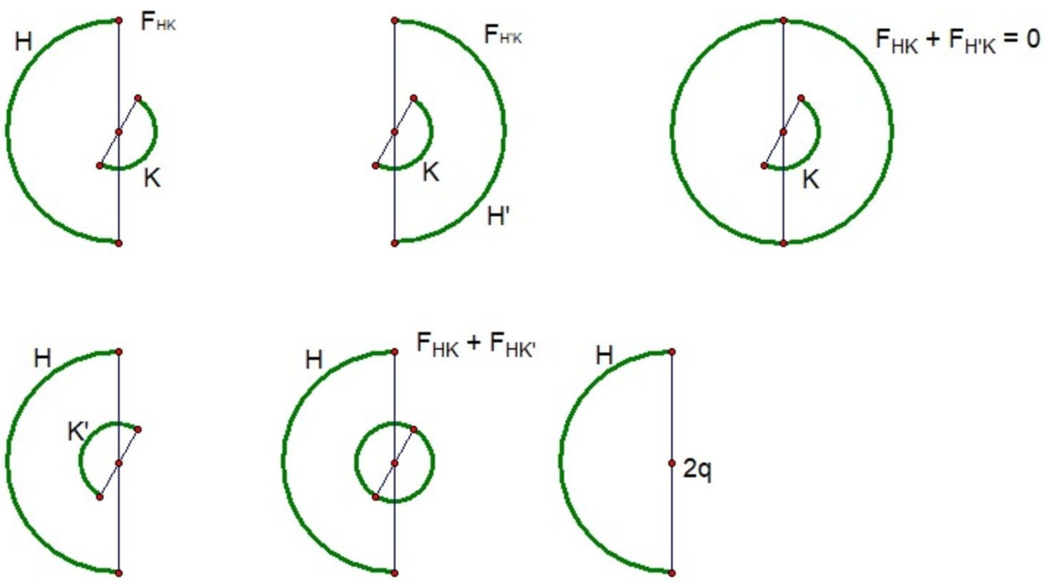
4. Find the force (with direction) of repulsion between the two uniformly charged insulating concentric hemispherical shells shown in the given figure.

The radius of larger shell = R

Radius of smaller shell = r.

Now consider that the smaller sphere starts expanding, and the charge on it remains uniformly distributed at any given time. What happens when r becomes bigger than R? Why?

Call the larger hemispherical shell *H* and the smaller shell *K*. Imagine a second uniformly charged large hemispherical shell *H'* which, together with *H*, forms a complete uniformly charged spherical shell with total charge 2Q. Let F_{HK} be the electrostatic force between *H* and *K*, and let $F_{H'K}$ be the electrostatic force between *H'* and *K*. Since the electric field inside a hollow uniformly charged hemisphere is zero, we deduce that $F_{HK} + F_{H'K} = 0$.



If we now imagine a second uniformly charged small hemisphere *K'*, which, together with *K*, forms a complete uniformly charged spherical shell of charge 2q, and let $F_{HK'}$ be the electrostatic force between *H* and *K'* then, by symmetry, $F_{HK'} = -F_{H'K} = F_{HK}$. Thus $F_{HK} = \frac{1}{2}(F_{HK} + F_{HK'})$, and this last is the electrostatic force between *H* and the entire smaller hemispherical sphere. Since the electric field of a uniformly charged spherical shell is, outside that shell, the same as the electric field of a point charge (of the same charge as that on the shell) at the centre of the shell, we deduce that F_{HK} is equal to $\frac{1}{2}G_{2q} = G_q$, where G_u is the electrostatic force between *H* and a point charge *u* at the centre of the hemispherical shell *H*.

It is thus, standard calculation now that

$$|F_{HK}| = |G_q| = \frac{Qq}{8\pi\varepsilon_0 R^2}$$

with the force acting along the line of symmetry of the larger hemispherical shell.

The problem with R=r is the fact that the field is not defined due to a hemisphere at its own surface. The charge distribution on its surface causes a sudden change in the field which directs the force along the other circle as we increase it's radius from less than R to greater than R.

Round 3: Video Round:

1. THE DANCING HANDLE:

https://youtu.be/1n-HMSCDYtM

This is a video of the installation handle on Space-DRUMS in free floating rotation. It rotates for sometime along an axis and then suddenly flips its axis of rotation. Will this happen under gravity as well? Explain.

Solution :

Intermediate axis theorem

This can be explained on the basis of the intermediate axis theorem.
There are 3 principal axes of rotation. Rotation about the first and third axis is stable but it is not so about the second axis.
That is why small disturbances about the other axes may cause it to flip.
The effect occurs whenever the axis of rotation differs slightly from the second principal axis.it does not require gravity or air resistance.

2. A THOUSAND DROPLETS:

https://youtu.be/jra7Tg2m5IY

Mr. Bean put a drop of mixture of water, isopropyl alcohol and food coloring on the surface of a bath of sunflower oil. The drop begins to fragment into multiple droplets and vanishes over time leaving a number of droplets in its place. Given the genius Mr. Bean is,he tries the experiment quite a number of times with different concentrations of alcohol and food coloring. What should he expect?

Solution:

The Marangoni effect

The phenomenon can be explained on the basis of the Marangoni effect.The Marangoni effect (also called the Gibbs–Marangoni effect) is the mass transfer along an interface between two fluids due to a gradient of the surface tension.
The alcohol evaporates from the rim giving rise to a surface tension gradient. There is lower surface tension towards the centre of the mother droplet. The particles are pulled outwards where the surface tension is higher and form little globules as they break away from the mother droplet.
If the alcohol concentration is too low, it cannot generate a sufficient surface tension gradient, so the Marangoni effect cannot be observed clearly. There is no role of the food color in this experiment. It's just to make it look beautiful :p)

3. THE MANY-SUN PROBLEM:

https://youtu.be/qw2yHQapSE0

The phenomenon in the previous video is called Parahelio. It occurs in Sweden and some other countries in the winter season. The sky appears quite gorgeous during a Parahelio. Ironman caught a glimpse of this while pursuing Ultron in Stockholm which dazzled him. Being Ironman, he wants to see it from other planets. So he invests heavily in interplanetary travel and makes a spaceship that can travel anywhere within the solar system. Which planet should he travel to first? Justify your answer.

Solution:

Parahelio

Sun dogs are commonly caused by the refraction and scattering of light from plate-shaped hexagonal ice crystals either suspended in high and cold cirrus or cirrostratus clouds, or drifting in freezing moist air at low levels as diamond dust.[2]The crystals act as prisms, bending the light rays passing through them with a minimum deflection of 22°. As the crystals gently float downwards with their large hexagonal faces almost horizontal, sunlight is refracted horizontally, and sun dogs are seen to the left and right of the Sun. Larger plates wobble more, and thus produce taller sundogs.It is possible to predict the forms of sun dogs as would be seen on other planets and moons. Mars Might have sun dogs formed by both water-ice and CO2-ice. On the gas giant planets — Jupiter, Saturn, Uranus and Neptune—other crystals form clouds of ammonia, methane, and other substances that can produce halos with four or more sun dogs.

4. POLE DANCING LIQUID:

https://youtu.be/IPKgsra9N8s

It's 29th Century and a new trend has hit the entire globe, the "Pole dancing Liquid" has got billions of fans on the planet (Yes, The Earth still exists!) . Everybody wants to know why does this liquid go crazy. As being the genius, people approach you for the solution. Can you help them out?

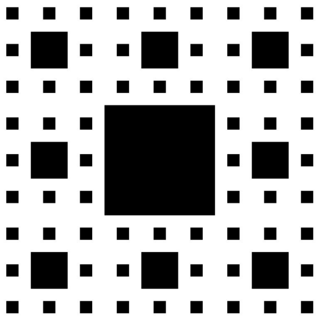
Solution:

The Weissenberg effect:

The liquid being rotated is a viscoelastic fluid. So it basically tries to resist changes induced by an external force. You can imagine each polymer as a spring. Whenever the rod tries to push the liquid away, the spring tries to regain it's shape by pulling in. So the springs squeeze the liquid that is coming out after being thrown away by the rod. This forces the liquid to rise along the rod.

Round 4: Buzzer Round:

1. Calculate the moment of inertia of a Sierpinski carpet with mass *M* and length *L* about the axis that is perpendicular to it and passes through the center. The construction of the Sierpinski carpet begins with a square of side length L. The square is cut into 9 congruent subsquares in a 3-by-3 grid, and the central subsquare is removed. The same procedure is then applied recursively to the remaining 8 subsquares, ad infinitum.



A figure of an intermediate step of the construction.

Solution: We know, by dimensional analysis that the moment of inertia about the desired axis will be given by kML^2 where k is some dimensionless constant.
It can be seen that placing 8 Sierpinski carpets appropriately gives us a bigger Sierpinski carpet of length 3L. However, the mass of this is 8M. (Not 9M)
Thus, the moment of inertia of this Sierpinski carpet about the similar axis is $72kML^2$.
This same moment of inertia can be calculated in terms of the smaller 8 components by using parallel axis theorem and adding them.

This gives us:

$$72kML^2 = 4(kML^2 + ML^2) + 4(kML^2 + M(\sqrt{2}L)^2)$$
$$\Rightarrow 64kML^2 = 12ML^2$$
$$\Rightarrow k = \frac{3}{16}$$

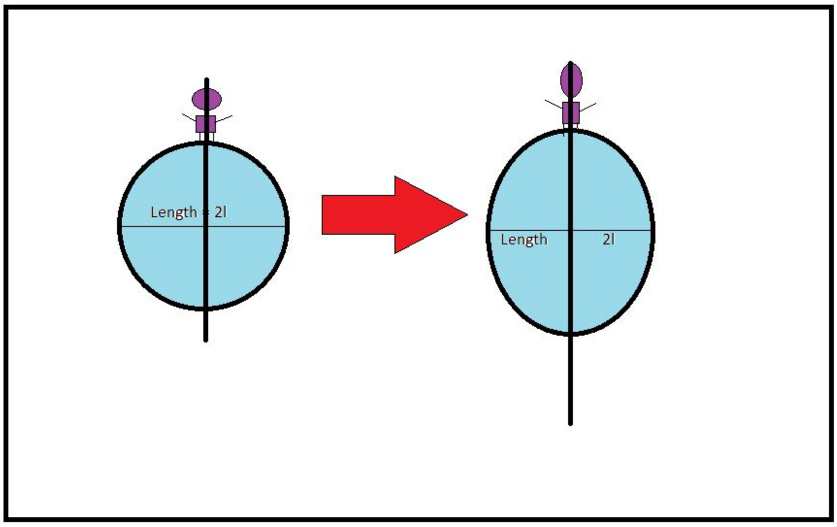
The first term was due to the contribution of the 4 squares placed adjacent to the middle hollow gap and the second one was due to the 4 squares placed at the corners.

2. Circles on the ice, a puck slides with speed v on frictionless ice. The surface is “level”, in the sense that it is perpendicular to the direction of a hanging plumb bob at all points. Show that the puck moves in a circle, as seen in the earth’s rotating frame. What is the radius of the circle? What is the frequency of the motion? Assume that the radius of the circle is small compared to the radius of the earth.

Solution: By construction, the normal force from the ice exactly cancels all effects of the gravitational and centrifugal forces in the rotating frame of the earth (because the plumb bob hangs in the direction of the “effective gravity” force, which is the sum of the gravitational and centrifugal forces). We therefore need only concern ourselves with the Coriolis force. This force equals $F_{cor} = -2m\omega \times v$
Let the angle down from the north pole be θ (we assume the circle is small enough so that θ is essentially constant throughout the motion). Then the component of the Coriolis force that points horizontally along the surface has magnitude f = 2mωv cos θ and is perpendicular to the direction of motion. (The vertical component of the Coriolis force will simply modify the required normal force.) Because this force is perpendicular to the direction of motion, v does not change. Therefore, f is constant. But a constant force perpendicular to the motion of a particle produces a circular path. The radius of the circle is given by

$$2m\omega v\cos\theta = \frac{mv^2}{r} \quad \Longrightarrow \quad r = \frac{v}{2\omega\cos\theta}. \qquad \omega' = \frac{v}{r} = 2\omega\cos\theta.$$

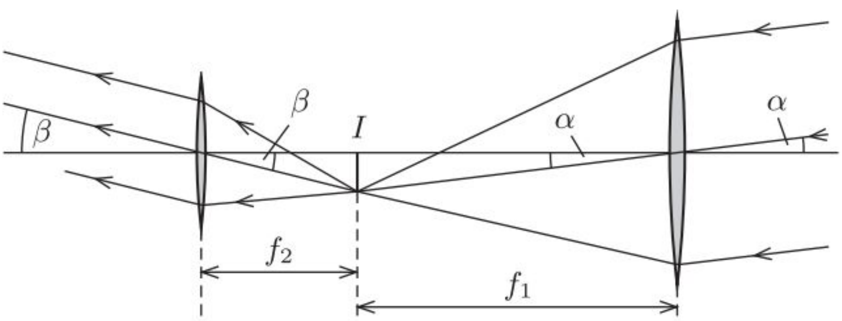
3. You and your friend had a bet that if you could spin yourself about the length of your body (the axis of rotation being the vertical line passing through your center of mass and parallel to the length of your body) for two hours, then he will give you a treat. As you were doing the impossible, Earth and all it’s residents are elongated along an axis through the center of the earth passing through the length of your body by the aliens (Yes, they have arrived!!!) by a factor of 1.25(the cross-section remains the same). Assume that the distribution of mass of your body (and of the entire earth) remains the same except along the length of your body. (of course, the mass density increases). What will be the change in your angular velocity?Justify.



Solution: The angular velocity remains the same. One can calculate easily that moment of inertia remains the same (Strech theorem). And since the angular momentum remains conserved(if one imagines the external force, then this force passes through the axis of rotation and hence has no torque about the axis of rotation), the result follows.

4. How many times 'brighter' is an image of the Moon when looked at through a telescope rather than with the naked eye? And what about the stars?Reason.

Solution: Consider, for example, an astronomical or Keplerian telescope, in which the image is formed by two convex lenses. The image of a very distant object, formed by the objective lens, lies almost exactly in the focal plane of that lens. If we use an ocular (eyepiece) lens to look at this image, its focal plane is made to almost coincide with the position of the image; this is because our eye is naturally accommodated to 'infinity', i.e. to view objects as if they were very distant. Let us denote the focal lengths of the objective and ocular lenses by f1 and f2, respectively.



It can be seen from Fig. 1, which is strongly distorted, that the angular magnification of the telescope (the factor by which the angular diameter of the Moon's circular face appears to be magnified) is

$$M_{\text{ang}} = \frac{\beta}{\alpha} \approx \frac{\tan\beta}{\tan\alpha} \approx \frac{(I/f_2)}{(I/f_1)} = \frac{f_1}{f_2}.$$

Next, consider how much greater is the light energy that enters our eye via the telescope compared to what it would be if we looked directly at the Moon. Assume that the diameter d_2 of our pupil is the same in both cases and that it is smaller than the diameter of the telescope's ocular lens. It can be seen from Fig.2 (which shows only almost-parallel light rays, and omits those needed for image construction) that the amount of light entering the eye when using a telescope, is a factor of $\left(\frac{d_1}{d_2}\right)^2 = \left(\frac{f_1}{f_2}\right)^2 = M_{\text{ang}}^2$ times larger than it would be without the telescope.

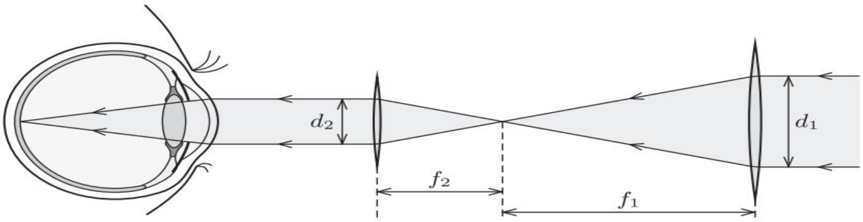


Fig. 2

Apparently, the Moon seems just as bright when looked at with the naked eye as it does through a telescope!

Our considerations are not valid for celestial bodies that subtend such a small visual angle that their images are not extended areas, but cover only a single photoreceptor in the retina (or a single pixel in a digital camera). In such cases, the telescope still increases the power of the incoming light, but it is not diluted by a size increase of the image; consequently, we observe the star to be brighter than it appears to the naked eye.

5. Consider an engine that drives a train initially at rest to velocity v and then to 2v. Assuming no heat loss, work done in the process is 2mv^2.
Now consider the same scenario as observed from an inertial frame moving at v in the same direction as the train. It's easy to see that work done by engine according to this frame of reference is mv^2. Explain as to where is the ambiguity.

Solution: Assume that the distance travelled in the first case is D. since work done by all the forces is equal to change in kinetic energy of the system, hence we can write the work done =f*D, where f is the friction force. In the second case, the observer on the moving frame observes ground beneath the train moving and work done in that process is f*vt and for the train, he observes a work done=f*(D-vt). Adding the two, we see that work done in the second case is the same as that in first.

6. How high could the tallest mountain on Earth be? And on Mars?(latent heat of melting of metals (200- 300 kJ kg-1) g_mars = 4ms^-2). Show your calculations.

Solution: Consider a 'box-shaped' mountain of average density p, base area A and height h.
In order to melt its bottom layer of thickness d and specific latent heat L, energy AdpL would be required. The total mass of the mountain is approximately Ahp, and the energy released if it sank a distance d would therefore be Ahpgd. The base of the mountain does not melt under its load if AdpL > Ahpgd, . i.e h< L/g
Approximating the required latent heat by the latent heat of melting of metals (200- 300 kJ kg-1), we estimate the maximum possible height of mountains on Earth to be 20-30 km. This is of the right order of magnitude. Allowing for the fact that the base of the mountain does not actually have to melt, but rather that the size of mountains is limited by the yield strength of their constituent materials, the estimated height of the highest mountains on Earth is surprisingly accurate.
Gravitational acceleration is significantly smaller on Mars than on Earth (gMars ~ 4 m s-2).
Therefore mountains, consisting of similar rocks, could be higher on Mars than on Earth. Indeed, the highest mountain on Mars, Mons Olympus, is 26 km high!