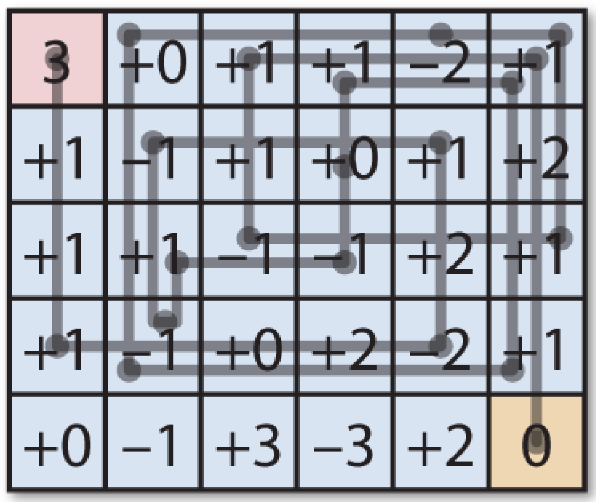
Logic GC Round 2 solutions

October 15, 2019

Ans 1:
Here's the 33-step solution, in the process of which ten squares are visited twice:



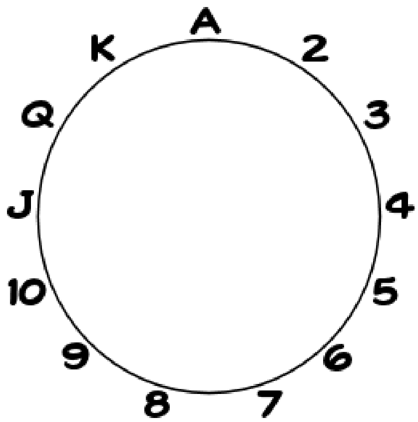
numbering:

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

The sequence of cells is as follows:
1 19 23 11 8 20 14 16 10 4 6 24 20 2 5 6 18 15 3 6 30

Ans 2:

There are five cards and only four suits, so at least two of the five cards given to Naomi must be the same suit. Naomi chooses a card that shares a suit with one of the other cards and puts another card of the same suit as the topmost of the four cards she hands to Natasha. The suit of the top card indicates the suit of Naomi's card. Naomi chooses the card of the duplicate suit to keep based on this diagram:

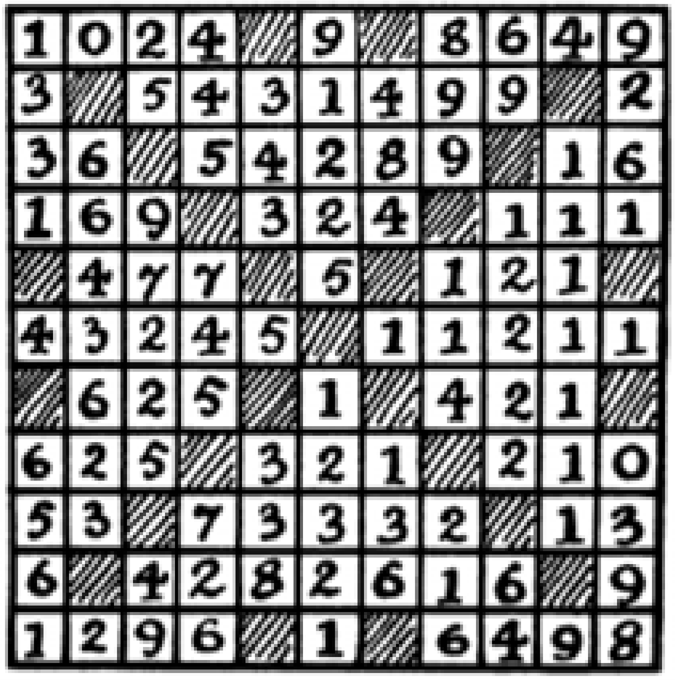


Naomi chooses a card that is between 1 and 6 positions to the right (clockwise) of the topmost of the four cards she hands to Natasha. This narrows down the possibilities for Naomi's card to one of 6 cards. The two girls have numerically ranked the 52 cards from high (ace) to low (two) with ties broken by suit as follows: spade, heart, diamond, then club. The three bottom cards are assigned a position of high (H), middle (M), or low (L) based off their rank compared to the other two cards. There are 6 ways to arrange the three bottom cards: LMH, LHM, MLH, MHL, HLM, HML which respectively indicate that Naomi's card is 1, 2, 3, 4, 5, or 6 positions to the right of the top card. That's how Natasha knows the rank of Naomi's card, based off the formation of how the bottom three cards are arranged.

For example, if the five cards the spectator gives to Naomi are the 9 of spades, 2 of spades, 5 of diamonds, 8 of clubs, and 8 of hearts. Naomi would choose to keep the 2 of spades and the 9 of spades would be the topmost of the four cards she hands to Natasha. This tells Natasha that Naomi's card is a spade and that the rank is 1 to 6 positions to the right of the 9 (i.e. 10, J, Q, K, A, or 2) as per the above diagram. The bottom three cards would be arranged in the order HML as per their system to indicate that Naomi's card is 6 positions to the right of the 9. So Naomi would hand Natasha the cards in the following order (from top to bottom): 9 of spades, 8 of hearts, 8 of clubs, 5 of diamonds.

Ans 3:

The difficulty is to know where to start, and one method may be suggested here. In reading the clues across, the most promising seems to be 18 across. The three similar figures may be Ill, 222, 333, and so on. 26 down is the square of 18 across, and therefore 18 across must be either III or 222, as the squares of 333, 444, etc., have all more than five figures. From 34 across we learn that the middle figure of 26 down is 3, and this gives us 26 down as the square of Ill, i.e., 12321. We now have 18 across, and this gives us 14 down and 14 across. Next we find 7 down. It is a four-figure cube number ending in 61, and this is sufficient to determine it. Next consider 31 across. It is a triangular number-that is, a number obtained by summing 1,2,3,4,5, etc. 210 is the only triangular number that has one as its middle figure. This settles 31 across, 18 down, 21 down, and 23 across. We can now get 29 across, and this gives us 30 down. From 29 down we can obtain the first two figures of 15 across, and can complete 15 across and 29 down. The remainder can now be worked out.



Ans 4:
Let x be the answer we want, the number of drops required.
So if the first egg breaks maximum we can have x-1 drops and so we must always put the first egg from height x. So we have determined that for a given x we must drop the first ball from x height. And now if the first drop of the first egg doesn't breaks we can have x-2 drops for the second egg if the first egg breaks in the second drop.
Taking an example, lets say 16 is my answer. That I need 16 drops to find out the answer. Lets see whether we can find out the height in 16 drops. First we drop from height 16,and if it breaks we try all floors from 1 to 15.If the egg don't break then we have left 15 drops, so we will drop it from 16+15+1 =32nd floor.
The reason being if it breaks at 32nd floor we can try all the floors from 17 to 31 in 14 drops (total of 16 drops). Now if it did not break then we have left 13 drops. and we can figure out whether we can find out whether we can figure out the floor in 16 drops.
Lets take the case with 16 as the answer
1 + 15 16 if breaks at 16 checks from 1 to 15 in 15 drops
1 + 14 31 if breaks at 31 checks from 17 to 30 in 14 drops
1 + 13 45 .....
1 + 12 58
1 + 11 70
1 + 10 81
1 + 9 91
1 + 8 100 We can easily do in the end as we have enough drops to accomplish the task
Now finding out the optimal one we can see that we could have done it in either 15 or 14 drops only but how can we find the optimal one. From the above table we can see that the optimal one will be needing 0 linear trials in the last step. So we could write it as
$(1+p)+(1+(p-1))+(1+(p-2))+\ldots\ldots\ldots+(1+0) >= 100$. Let 1+p=q which is the answer we are looking for $q(q+1)/2 >= 100$ Solving for 100 you get q=14. So the answer is: 14 Drop first orb from floors 14, 27, 39, 50, 60, 69, 77, 84, 90, 95, 99, 100... (i.e. move up 14 then 13, then 12 floors, etc) until it breaks (or doesn't at 100).

Ans 5:

1— 4	3+ 2	3— 1	144× 6	2— 5	3
5	1	4	3	2	30× 6
4— 2	6	3 3	4	1	5
2÷ 6	1— 3	3— 2	1— 5	4	1— 1
3	4	5	5— 1	6	2
4— 1	5	3÷ 6	2	12× 3	4

Ans 6:
for a+b+c, first two digits are obtained by multiplying a and b
next two digits are obtained by multiplying a and c
to obtain last two digits, find out (a*b)+(a*c)-c, and then swap the digits of the number obtained
For example,
for, 3+5+6
first two digits are 3*5 = 15
next two digits are 3*6 = 18, and
(3*5)+(3*6) - 6 = 27
so, the last two digits are 72

Ans 7:
As the divisor when multiplied by 7 produces only three figures we know the first figure in the divisor must be I. We can then prove that the first figure in the dividend must be I; that, in consequence of bringing down together the last two figures of the dividend, the last but one figure in the quotient must be 0, that the first and last figures in the quotient must be greater than 7, because they each produce four figures in the sum, and so on.

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124)12128316(97809
    1116
    ----
     968
     868
     ----
     1003
      992
     ----
       1116
       1116
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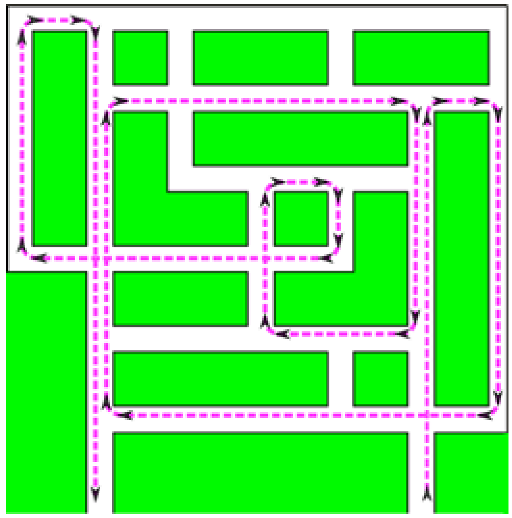
Ans 8:



Ans 9:
In a period of 400 years, the cycle of weekdays makes one complete cycle through the years (it cannot be a shorter period because of the odd leap year rule; it does not have to be a longer period because a period of 400 years has $400 \times 365 = 146000$ normal days plus 100 - 3 = 97 leap days, which makes 146097 days in total, which is divisible by 7). Therefore, we only have to look at the probability in a period of 400 years, for example the years 2001 up to 2400.
For a normal year, the distribution of the 13th days over the days of the week is as follows, if January 13th falls on weekday x:

x	x+1	x+2	x+3	x+4	x+5	x+6
2	1	1	3	1	2	2

For a leap year, the distribution of the 13th days over the days of the week is as follows, if January 13th falls on weekday x:

x	x+1	x+2	x+3	x+4	x+5	x+6
3	1	1	2	2	1	2

For subsequent years, these distributions shift cyclic one place to the right for normal years and two places to the right for leap years.
For three normal years followed by one leap year, we can now calculate the distribution of the 13th days:

x	x+1	x+2	x+3	x+4	x+5	x+6	
2	1	1	3	1	2	2	(normal year, not shifted)
2	2	1	1	3	1	2	(normal year, shifted one place to the right)
2	2	2	1	1	3	1	(normal year, shifted two places to the right)
+ 2	1	2	3	1	1	2	(leap year, shifted three places to the right)
8	6	6	8	6	7	7	(3 normal years and 1 leap year, not shifted)

For four normal years, we find:

x	x+1	x+2	x+3	x+4	x+5	x+6	
2	1	1	3	1	2	2	(normal year, not shifted)
2	2	1	1	3	1	2	(normal year, shifted one place to the right)
2	2	2	1	1	3	1	(normal year, shifted two places to the right)
+ 1	2	2	2	1	1	3	(normal year, shifted three places to the right)
7	7	6	7	6	7	8	(4 normal years, not shifted)

In four years, the latter two distributions shift five places cyclic to the right. So for a period of 16 years with 4 leap years, we find:

x	x+1	x+2	x+3	x+4	x+5	x+6	
8	6	6	8	6	7	7	(3 normal years and 1 leap year, not shifted)
6	8	6	7	7	8	6	(3 normal years and 1 leap year, shifted 5 places to the right)
6	7	7	8	6	6	8	(3 normal years and 1 leap year, shifted 10 places to the right)
+ 7	8	6	6	8	6	7	(3 normal years and 1 leap year, shifted 15 places to the right)
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27	29	25	29	27	27	28	(16 years with 4 leap years, not shifted)

For a period of 100 years with 24 leap years (the years 2001-2100, 2101-2200, and 2201-2300), we can now calculate the distribution of the 13th days:

x	x+1	x+2	x+3	x+4	x+5	x+6	
27	29	25	29	27	27	28	(16 years with 4 leap years, not shifted)
29	25	29	27	27	28	27	(16 years with 4 leap years, shifted 20 places to the right)
25	29	27	27	28	27	29	(16 years with 4 leap years, shifted 40 places to the right)
29	27	27	28	27	29	25	(16 years with 4 leap years, shifted 60 places to the right)
27	27	28	27	29	25	29	(16 years with 4 leap years, shifted 80 places to the right)
27	28	27	29	25	29	27	(16 years with 4 leap years, shifted 100 places to the right)
+ 8	7	7	6	7	6	7	(4 normal years, shifted 120 places to the right)
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172	172	170	173	170	171	172	(100 years with 24 leap years, not shifted)

For a period of 100 years with 25 leap years (the years 2301-2400), we can also calculate the distribution of the 13th days:

x	x+1	x+2	x+3	x+4	x+5	x+6	
27	29	25	29	27	27	28	(16 years with 4 leap years, not shifted)

29 25 29 27 27 28 27 (16 years with 4 leap years, shifted 20 places to the right)

25 29 27 27 28 27 29 (16 years with 4 leap years, shifted 40 places to the right)

29 27 27 28 27 29 25 (16 years with 4 leap years, shifted 60 places to the right)

27 27 28 27 29 25 29 (16 years with 4 leap years, shifted 80 places to the right)

27 28 27 29 25 29 27 (16 years with 4 leap years, shifted 100 places to the right)

+ 7 8 6 6 8 6 7 (3 normal years and 1 leap year, shifted 120 places to the right)

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171 173 169 173 171 171 172 (100 years with 25 leap years, not shifted)

Now we can calculate the distribution of the 13th days for the period 2001 up to 2400:

x x+1 x+2 x+3 x+4 x+5 x+6

172 172 170 173 170 171 172 (100 years with 24 leap years, not shifted)

170 173 170 171 172 172 172 (100 years with 24 leap years, shifted 124 places to the right)

170 171 172 172 172 170 173 (100 years with 24 leap years, shifted 248 places to the right)

+ 172 171 173 169 173 171 171 (100 years with 25 leap years, shifted 372 places to the right)

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684 687 685 685 687 684 688 (400 years, not shifted)

Since January 13, 2001 (x in the distribution) is a Saturday, we get the following probability for the 13th days over the days of the week:
Saturday:
684/4800
Sunday:
687/4800
Monday:
685/4800
Tuesday:
685/4800
Wednesday

687/4800
Thursday:
684/4800
Friday:
688/4800
Conclusion: the probability that the 13th of a certain month in a certain year is a Friday is the highest.

Ans 10:
A possible solution is:

Q1: Ask god B, "If I asked you 'Is A Random?', would you say ja?". If B answers ja, either B is Random (and is answering randomly), or B is not Random and the answer indicates that A is indeed Random. Either way, C is not Random. If B answers da, either B is Random (and is answering randomly), or B is not Random and the answer indicates that A is not Random. Either way, you know the identity of a god who is not Random.

Q2: Go to the god who was identified as not being Random by the previous question (either A or C), and ask him: "If I asked you 'Are you False?', would you say ja?". Since he is not Random, an answer of da indicates that he is True and an answer of ja indicates that he is False.

Q3: Ask the same god the question: "If I asked you 'Is B Random?', would you say ja?". If the answer is ja, B is Random; if the answer is da, the god you have not yet spoken to is Random. The remaining god can be identified by elimination.