BOUNCE OR POUNCE

House Rules

POUNCE= A team (who the Q is not directed to) decides to answer it, in under a time of min (Bounce, Pounce Period).

Bounce=the team (who the Q is directed to) answers the Q

If answered correctly: the Q is thrown.

If answered wrongly: the Q is bounced to the next team which hasn't pounced or is thrown once one cycle is completed.

Pounce Period= 90 seconds after the Q is released

After the pounce period, 60 additional seconds are given to the team

Join the Quiz



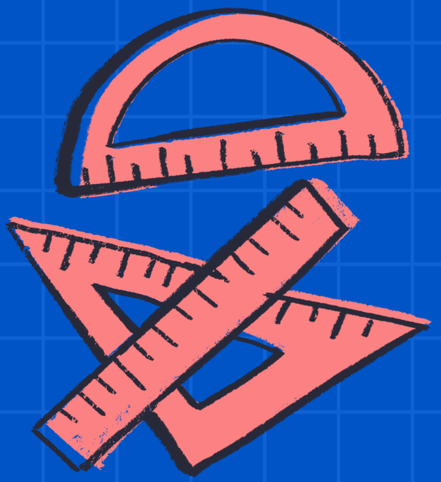
Use this link:
https://wayground.com/join?gc=724915

OR

Go to : https://wayground.com/join
Enter the code : 724915



Question 1.1



Q.9

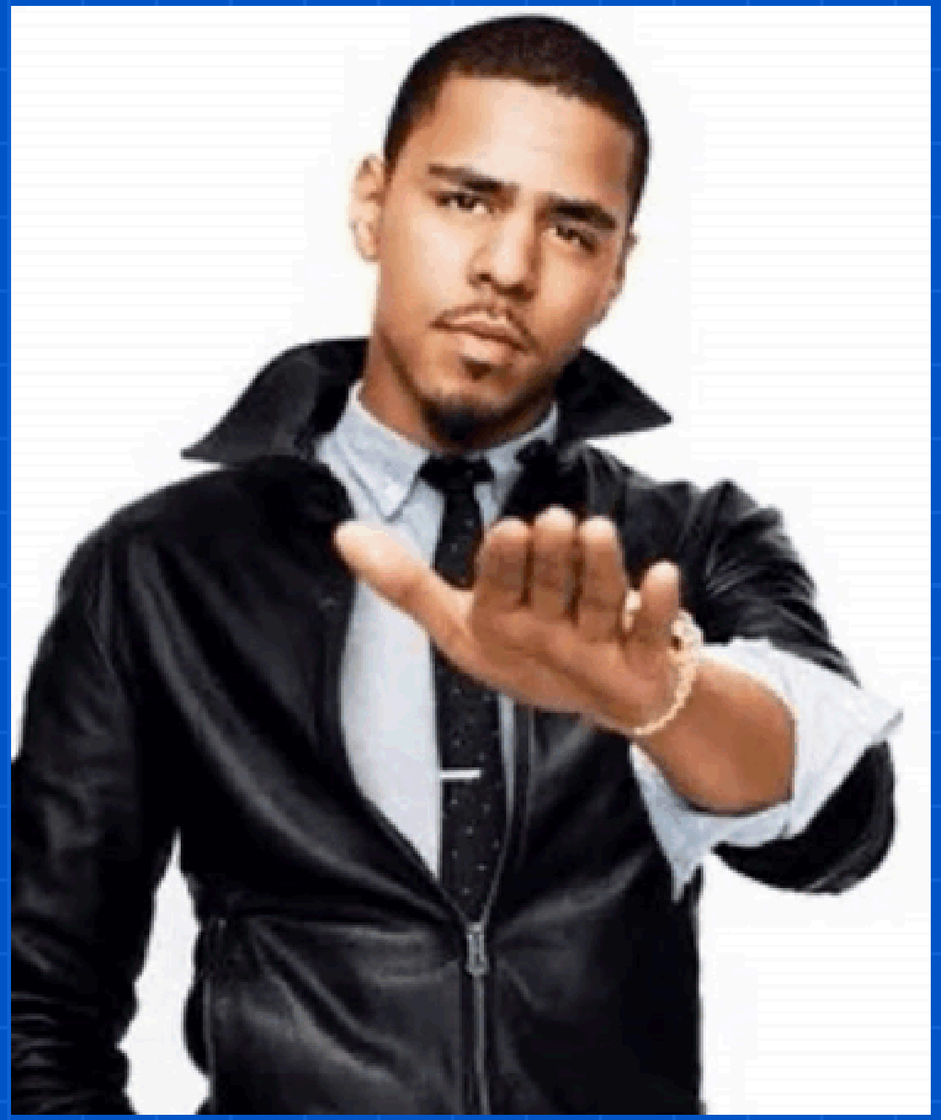
Let $S = \{1, 2, 3, \ldots, 10\}$. Consider the set

$$X = \{R : R \text{ is an equivalence relation on the set } S \text{ such that } R \text{ has exactly 42 elements}\}.$$

Then the number of elements in X is _____.

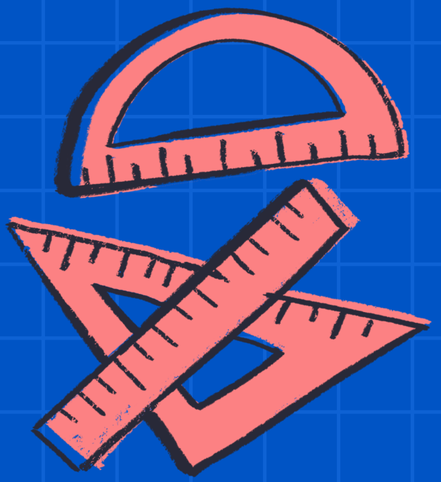


SAFETY SLIDE !!

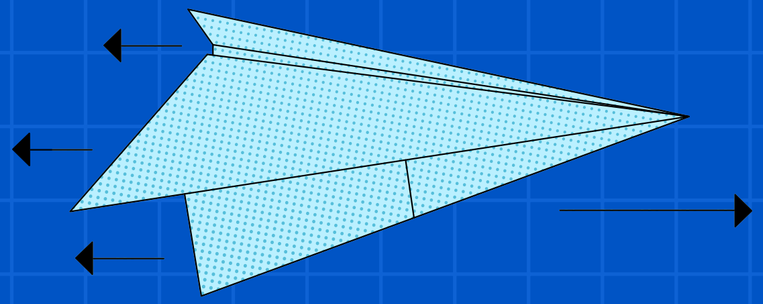




Question 1.1



How many subsets of $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ have no two successive numbers?

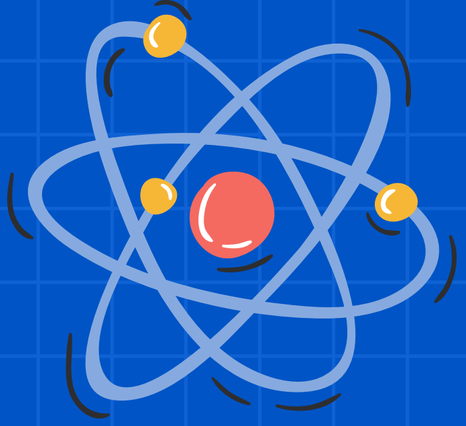


Answer

89

Solution Hint

Make a recurrence

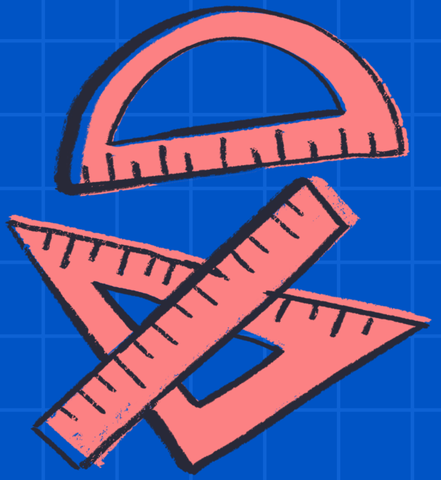


SAFETY SLIDE !!

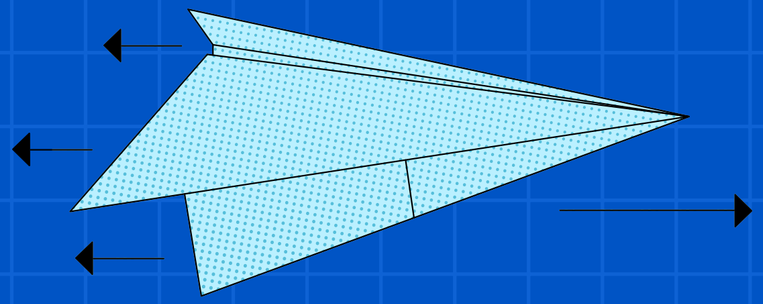




Question 1.2



How many triples (x, y, z) of different positive integers less than or equal to 8 satisfy $xy > z$, $xz > y$, and $yz > x$?

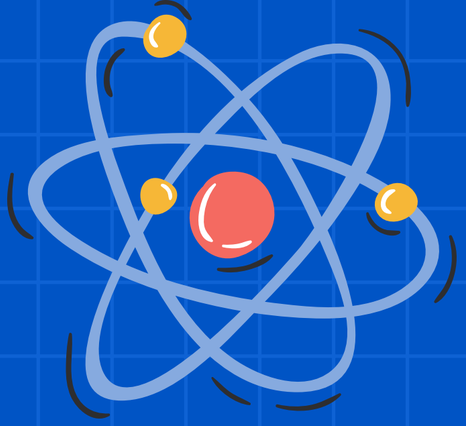


Answer

186

Solution Hint

Assume $x>y>z$

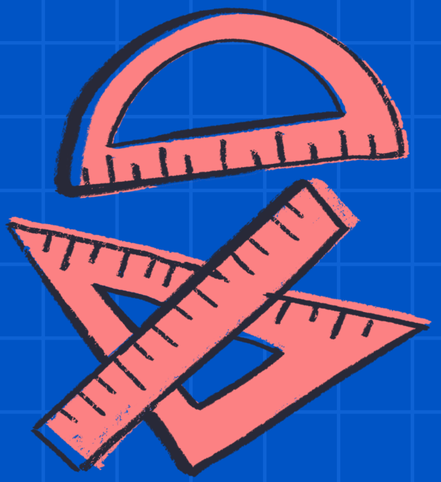


SAFETY SLIDE !!

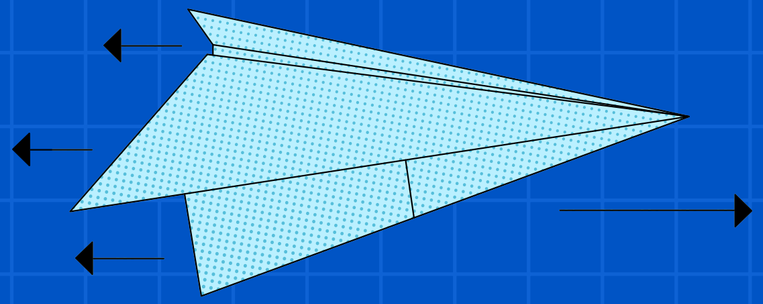




Question 1.3

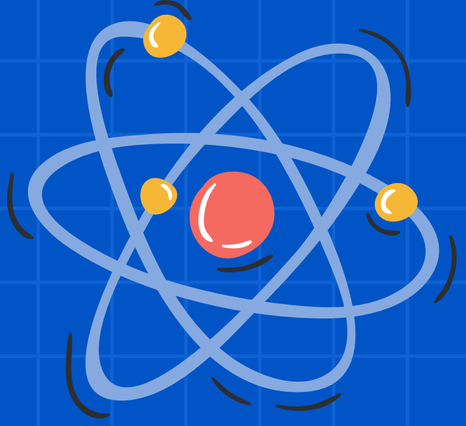


If a dice is rolled until 1, 2, and 3 occur consecutively, what is the probability that the dice is rolled an odd number of times?



Answer

216/431



Solution Hint

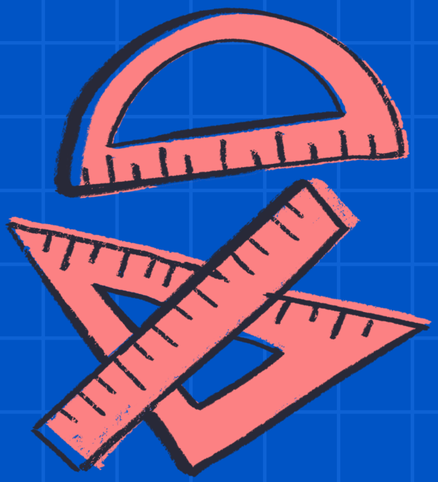
Make a probability tree

SAFETY SLIDE !!



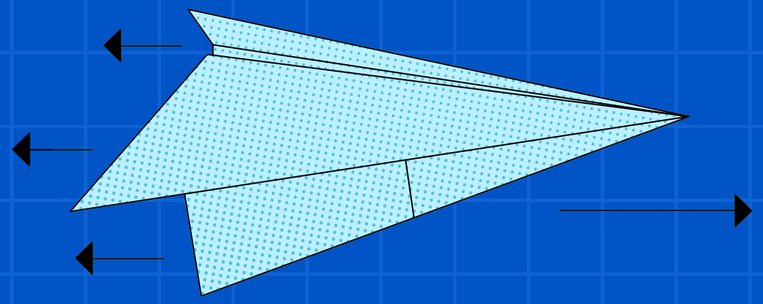


Question 1.4



Set $S = \{105, 106, \ldots, 210\}$. What is the minimum value of n such that any n-element subset T contains at least two non-relatively prime elements?

Hint: There are 46 primes between 1 and 200

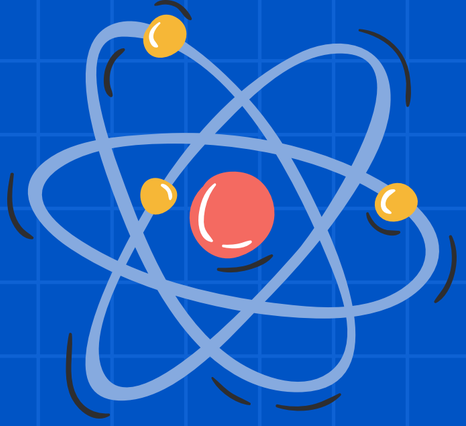


Answer

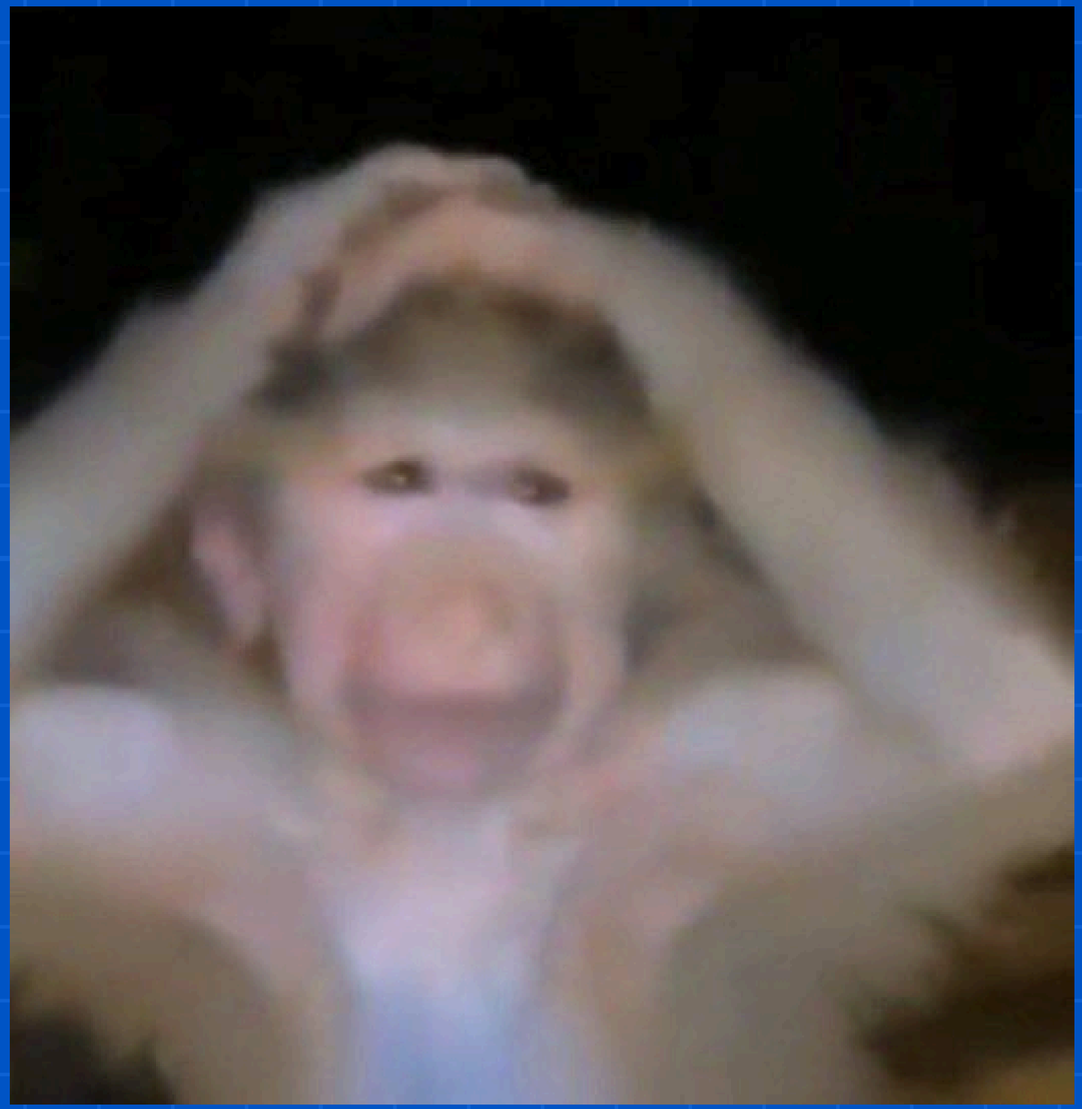
26

Solution Hint

Consider the worst case scenario

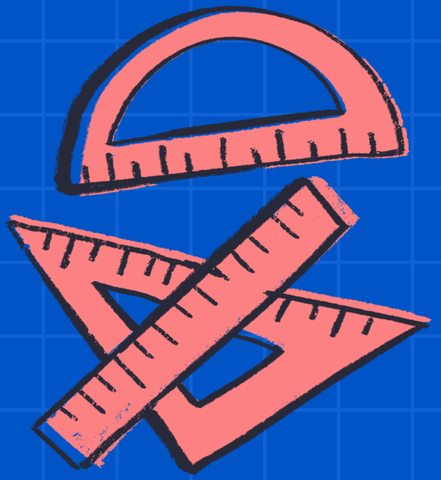


SAFETY SLIDE !!





Question 1.5

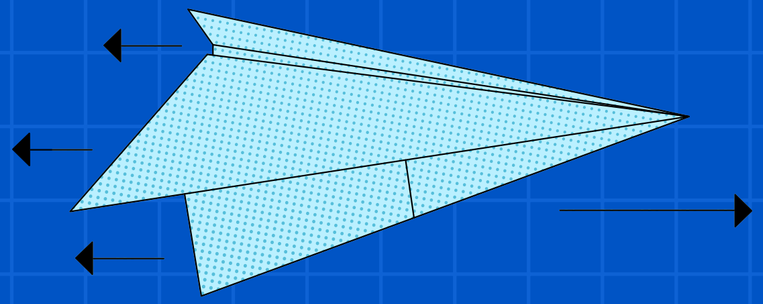


You have a hollow cube (side length $2r$) and a hollow tetrahedron centered at origin. The size of the tetrahedron is such that a sphere of radius $3r$ (centered at origin) perfectly fits inside it. The cube has surface mass density $\sigma_1 = 2$ and the tetrahedron has surface mass density $\sigma_2 = 7$.

Let S be defined as the integral $\int \frac{dm}{D^2}$ where D is the distance from origin. Find the ratio $\frac{S(\text{cube})}{S(\text{tetrahedron})}$.

SAFETY SLIDE !!



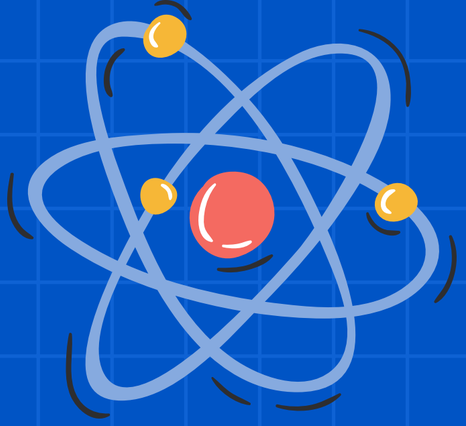


Answer

217

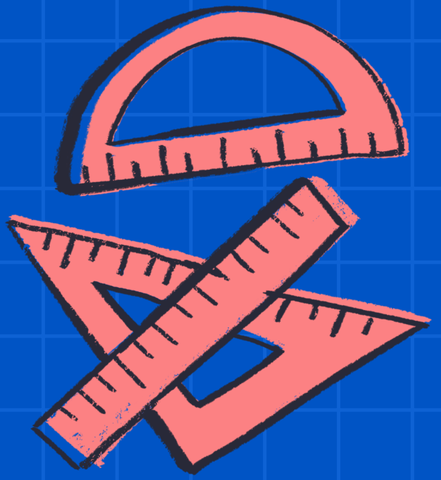
Solution Hint

Solid Angle!





Question 1.6



Let $x, y \in \mathbb{Z}^+$ satisfy the equation

$x^3 - y^3 = 2xy + 13$, call this equation P.

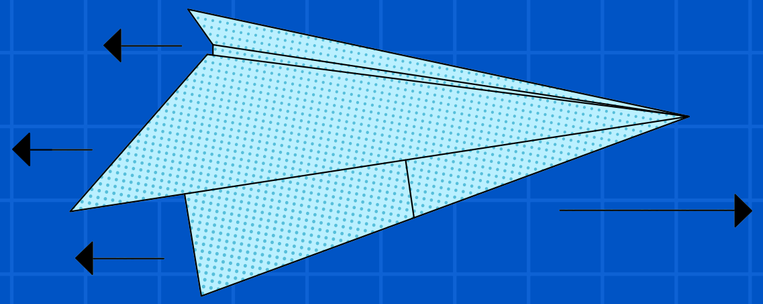
Call SET $A = \{x : P(x, y) = 0 \text{ for some } y\}$,

$B = \{y : P(x, y) = 0 \text{ for some } x\}$.

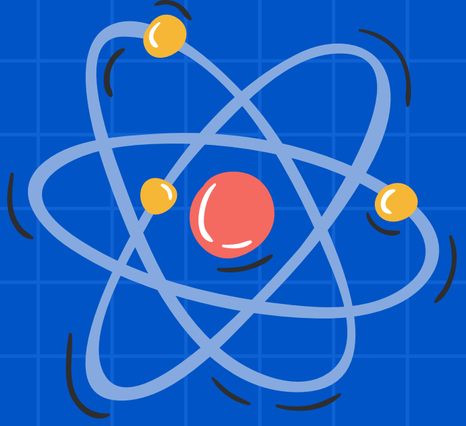
Find $\sum_{x \in A} x \cdot \sum_{y \in B} y$

SAFETY SLIDE !!





Answer



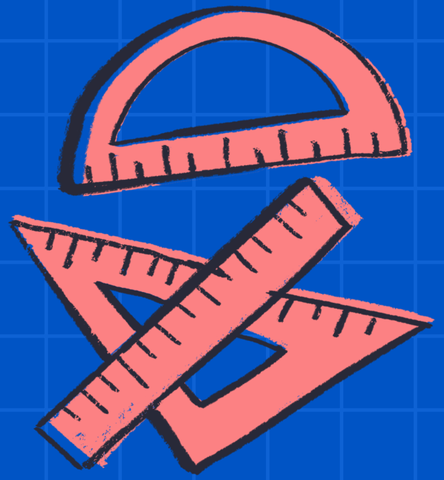
12

Solution Hint

try bounding the value of x-y



Question 2.1



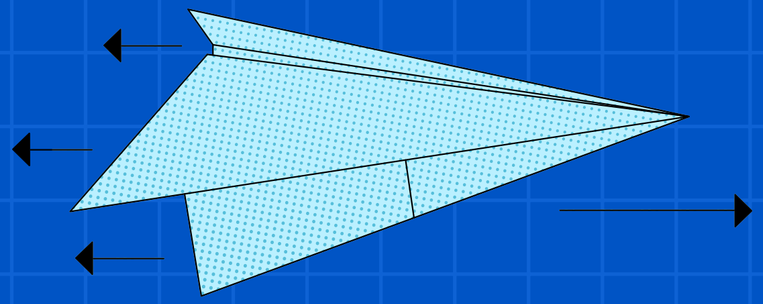
Evaluate the limit

$$\lim_{n\to\infty}\left(\int_0^5 e^{n(x^2+x+1)}\,dx\right)^{1/n}$$

SAFETY SLIDE !!

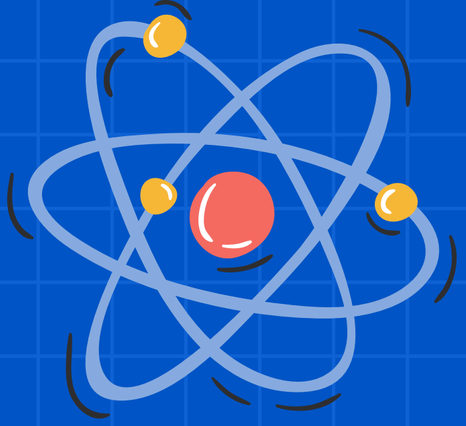
pizzas when?





Answer

e^31

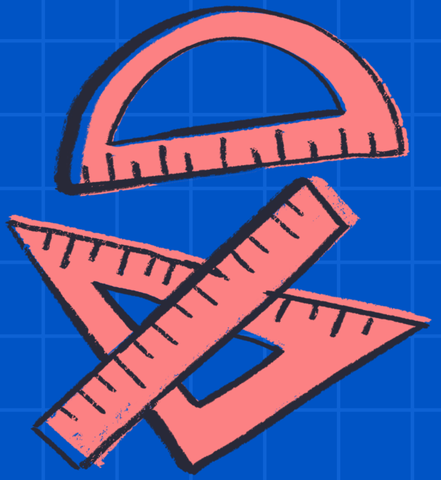


Solution Hint

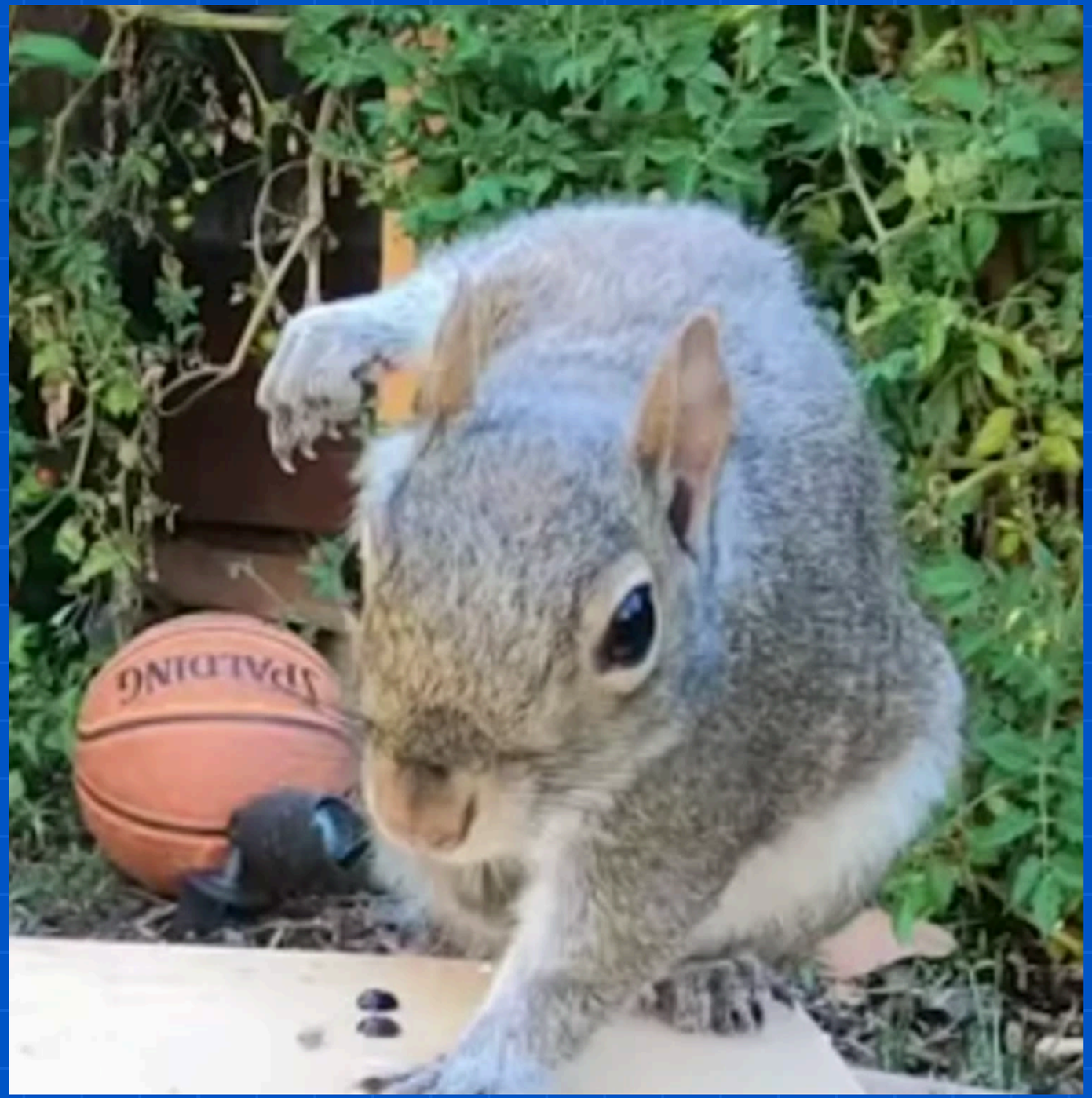
the last tiny portion of the integral dominates

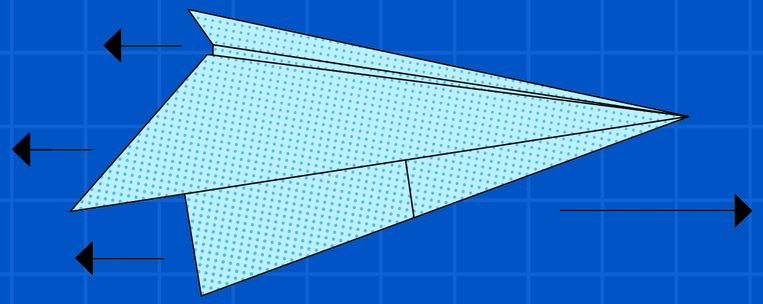


Question 2.2

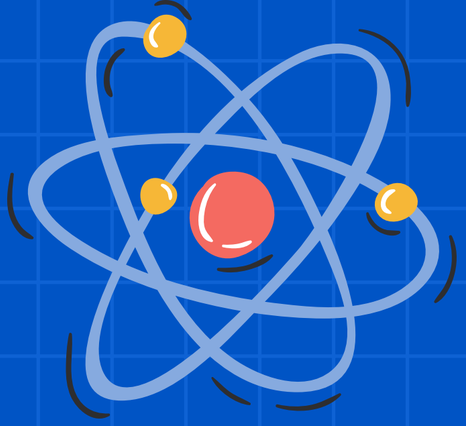


If $S = \sum_{k=1}^{25} \frac{(k^2)(4^k)}{(k+1)(k+2)}$, then S equals


SPALDING



Answer



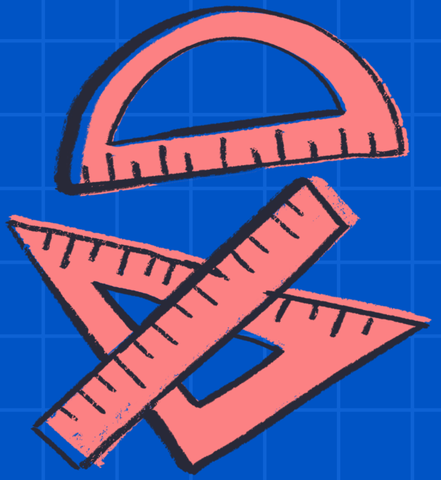
$$(2^{55} + 18)/27$$

Solution Hint

try writing k^2 in terms of $k+1$ and $k+2$



Question 2.3



Let $f(i) = \dfrac{i^2}{n^2} + \dfrac{i}{n}$ for $i = 1, 2, \ldots, n,$ and define

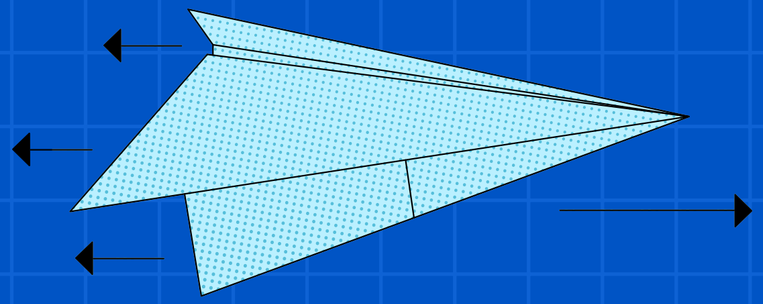
$$S_n = \min_{c \in \mathbb{R}} \sum_{i=1}^{n} \left| f(i) - c \right|.$$

Evaluate the limit

$$\lim_{n \to \infty} \frac{S_n}{n}$$

SAFETY SLIDE !!



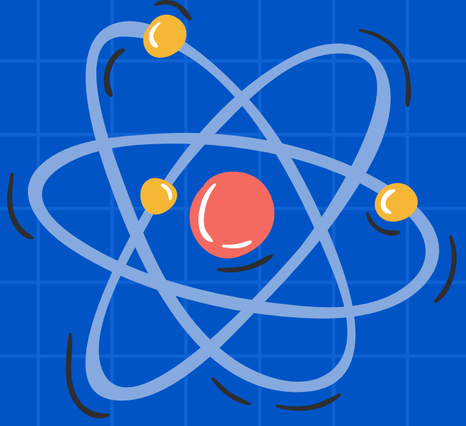


Answer

1/2

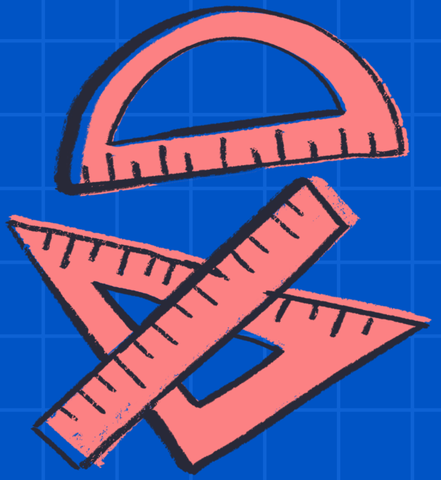
Solution Hint

Sn will be minimum at the median





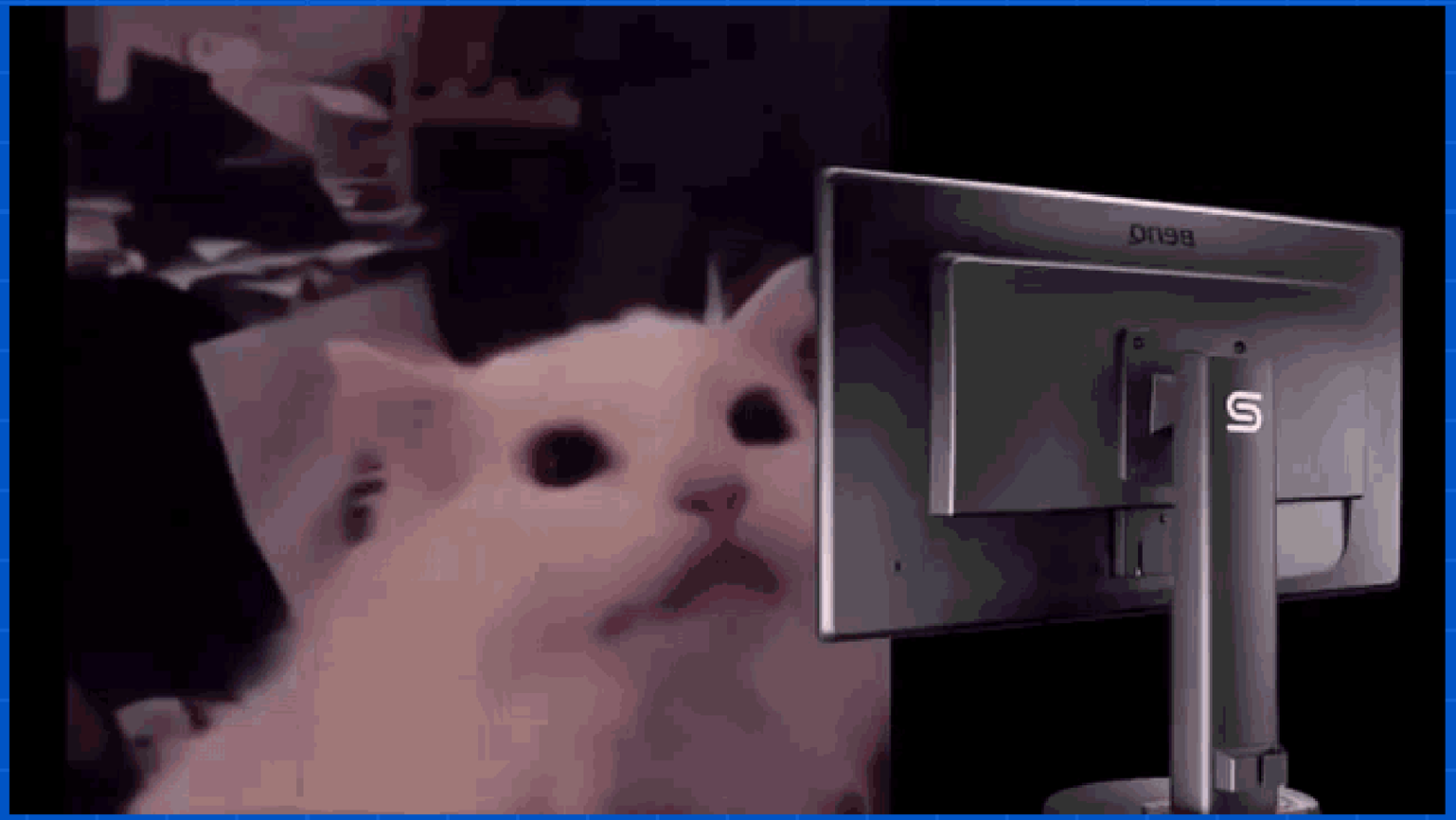
Question 2.4

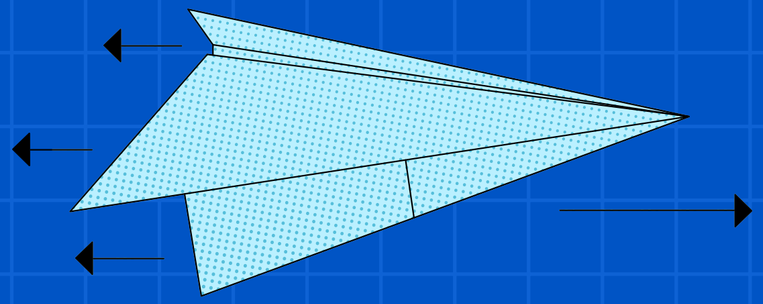


Find the number of integer solutions (x, y) to the equation

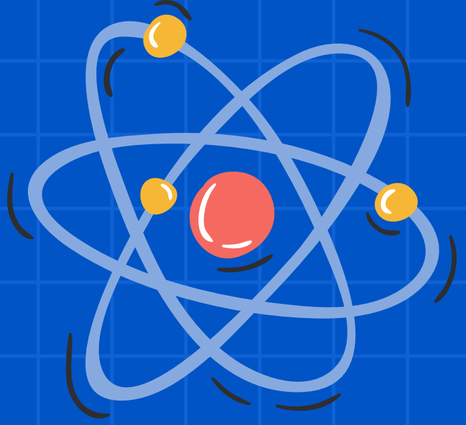
$$x^2 + xy + y^2 = 28$$

SAFETY SLIDE !!





Answer



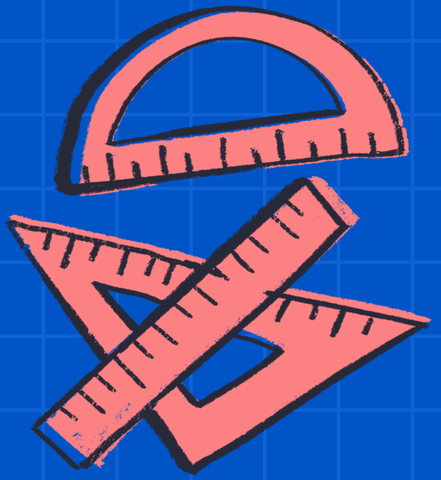
12

Solution Hint

Write it as a sum of squares and count



Question 2.5



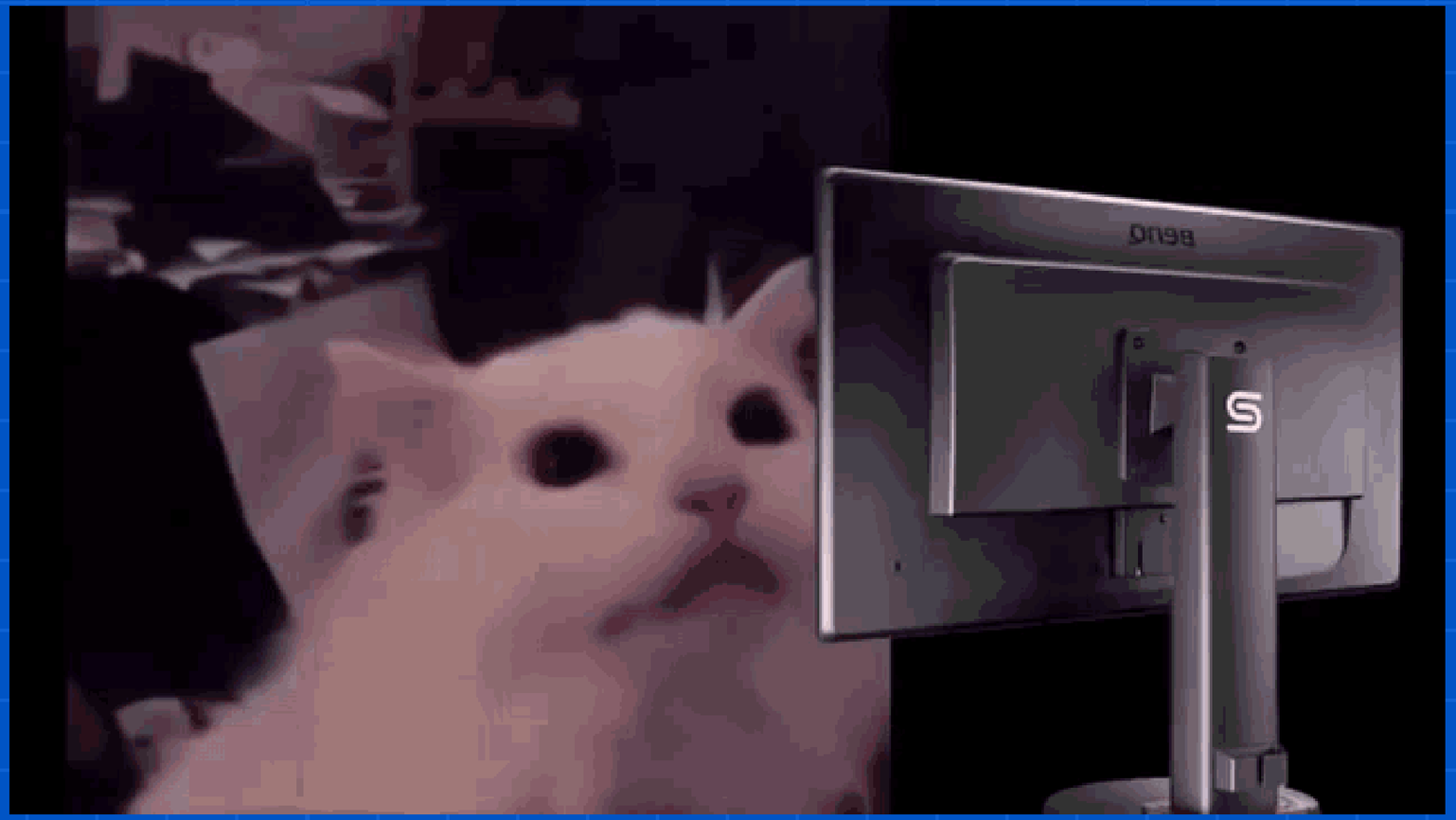
A bacterium can die, split into two, or split into three, with respective probabilities $\frac{1}{6}$, $\frac{2}{3}$, $\frac{1}{6}$.

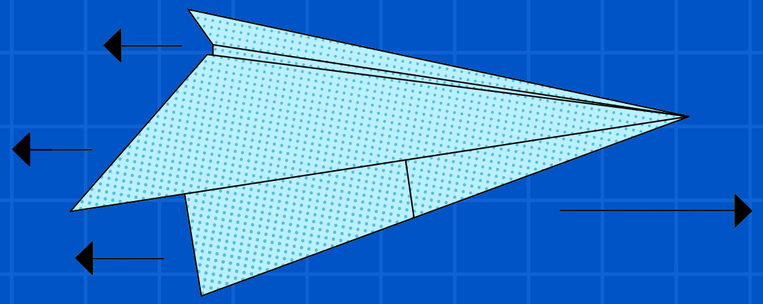
Each split produces exact copies, which independently follow the same process. Let the probability that the bacterial line survives forever be

$$\frac{m-\sqrt{p}}{n},$$

where p is a prime number. Find p.

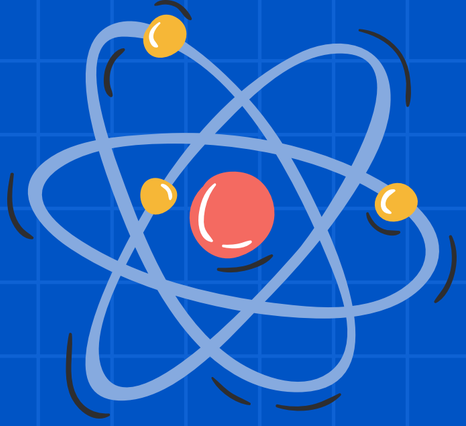
SAFETY SLIDE !!





Answer

29



Solution Hint

After the first move, each bacterium can be treated as independently

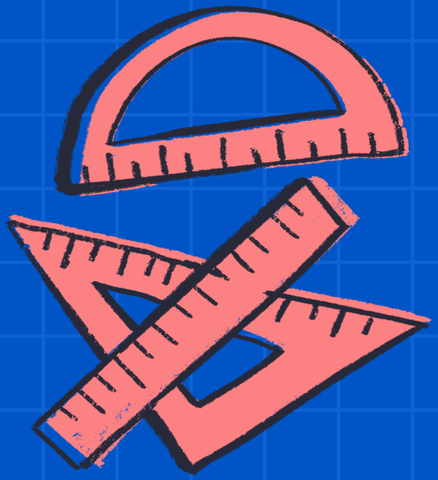
Onto the last question!

Me petting my cat for the last time before I sell him for RAM





Question 2.6



Suppose you have a large circular sheet of radius R.

It has a point marked at a distance d from the centre.

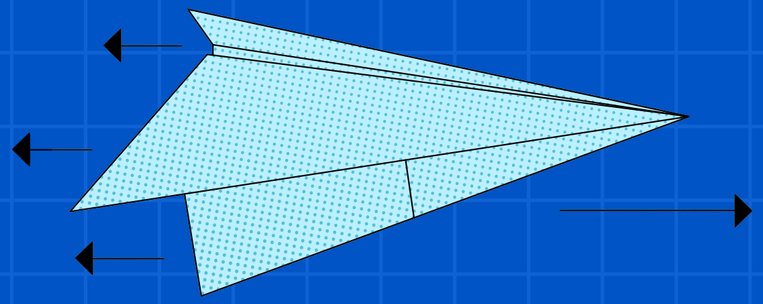
One by one, you start folding the sheet, mapping each point on the circumference onto the off-centre point, then unfolding it, making a clean crease. After having made infinitely many folds, you get a dense region through which no crease passes.

Find the area of this region, given $d = 6$, $R = 7$.

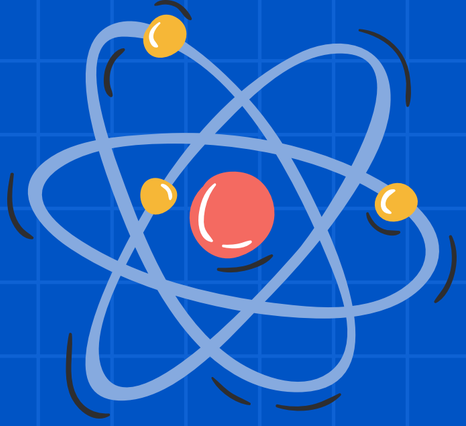
SAFETY SLIDE !!

pizzas when?





Answer



(7π/4)* sqrt(13)

Solution Hint

the region is an ellipse with foci as the center of the circle and the chosen point. Can you prove it?

ONTO THE NEXT ROUND!