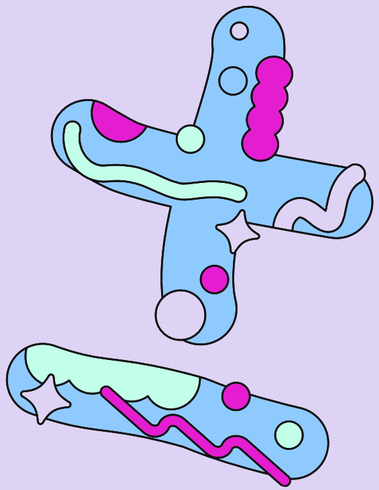
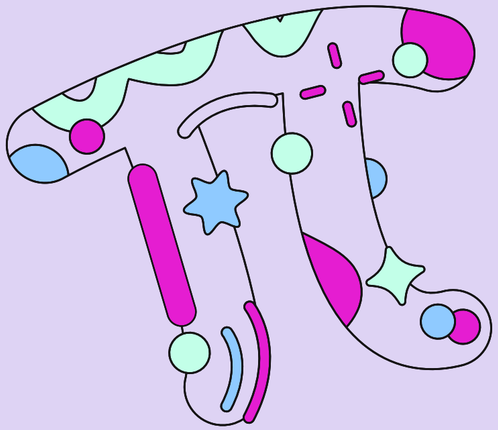

Points Blaster



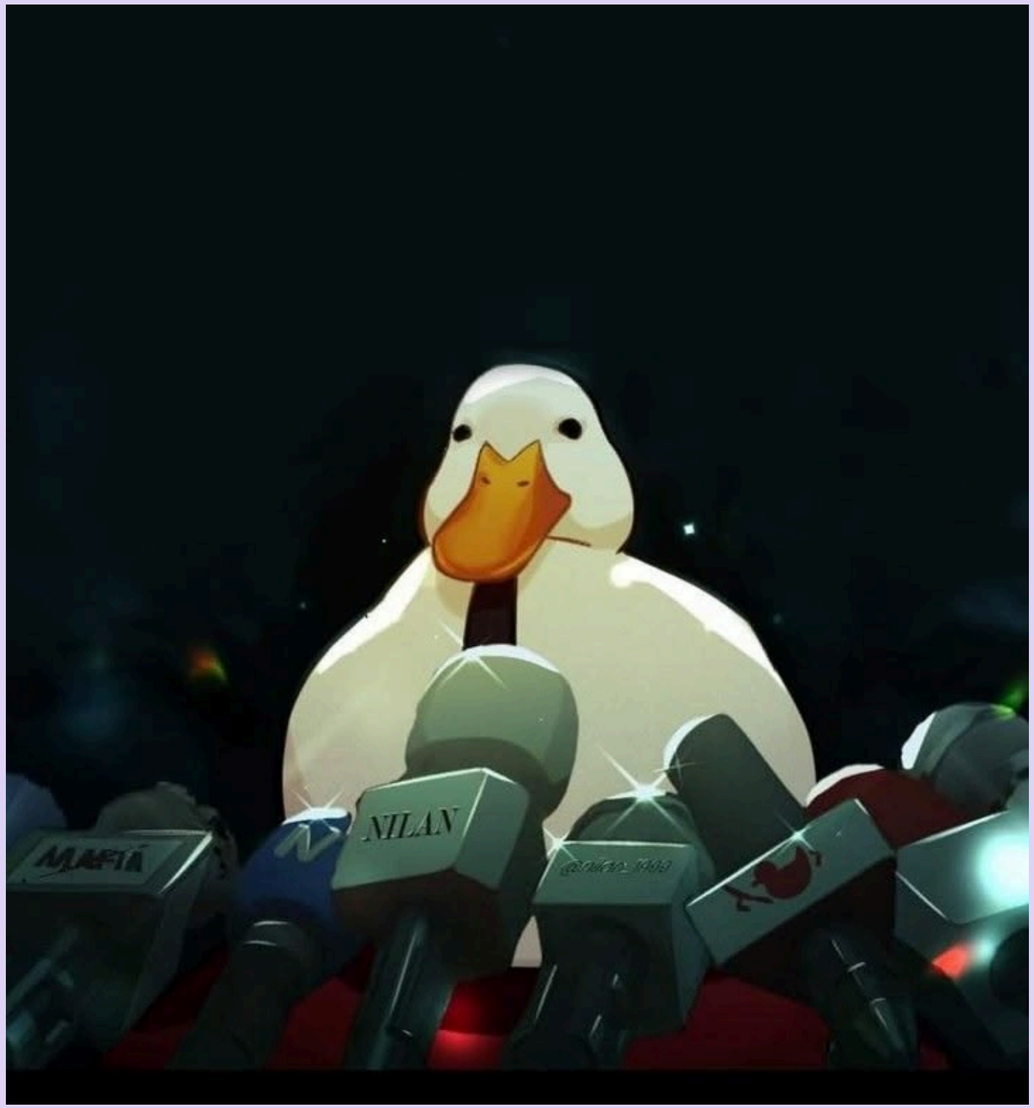
The Rules

- There will be a direct to each team twice (two rounds, total of 12 questions).
- We will show a printout of the question (with some part blanked out) and that team gets to bet anything from 100 to 200 points on it, call this x
- The other teams, with no information other than the amount that the first team has bet, gets to bet anything from x to $x - 100$.
- On getting it wrong, all teams get a negative of (whatever they bet)/2.





Direct to team 1...



Question 1

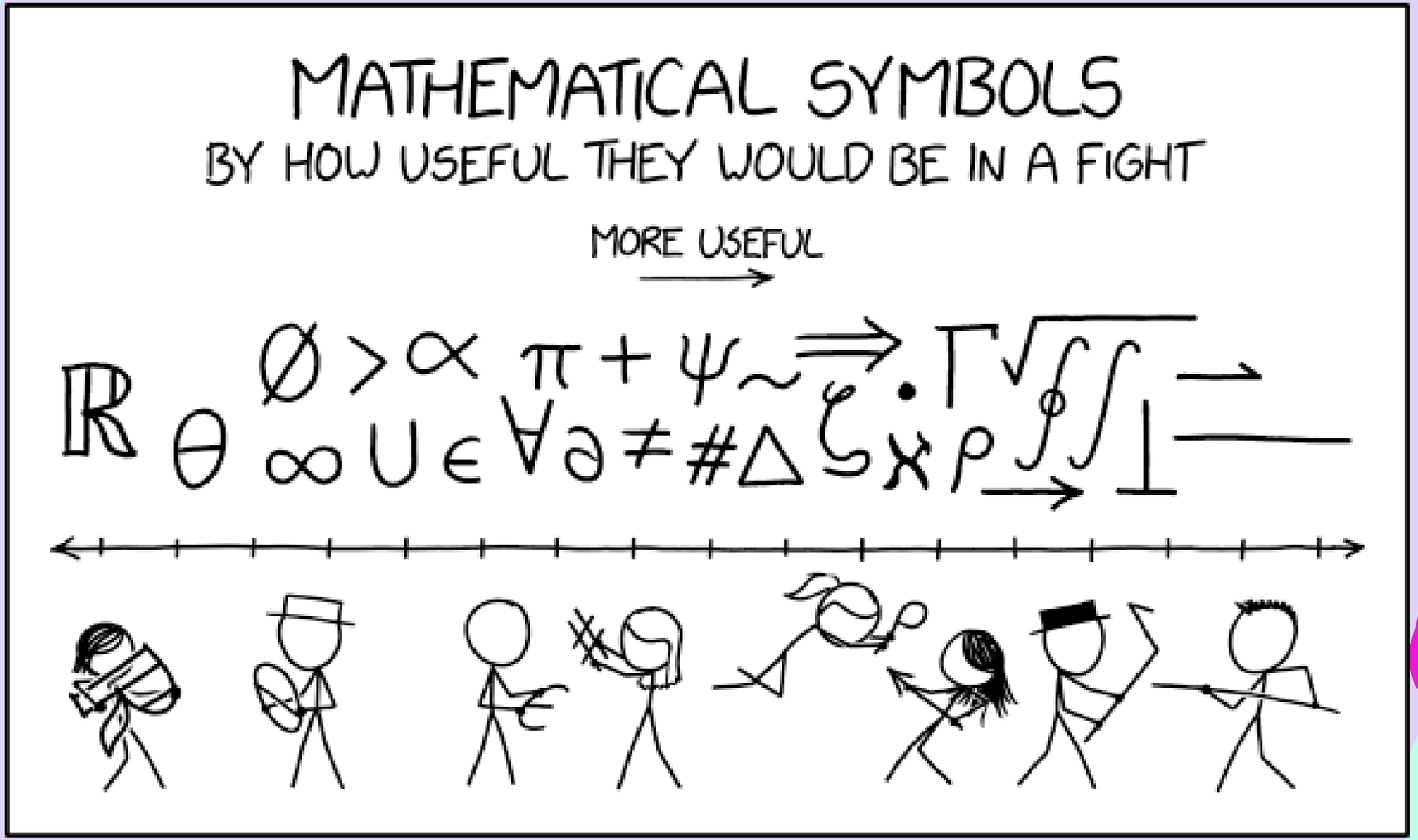
A frog hops along the number line according to the following rules.

- It starts at 0.
- If it is at 0, then it moves to 1 with probability $1/2$.
  and it disappears with probability $1/2$.
- For $n = 1, 2,$ or $3,$ if it is at $n,$ then it moves to $n + 1$
  with probability $1/4,$ it moves to $n - 1$ with probability $1/4,$
  and it disappears with probability $1/2$.

The probability that the frog reaches point 4 is $\frac{1}{x}$.

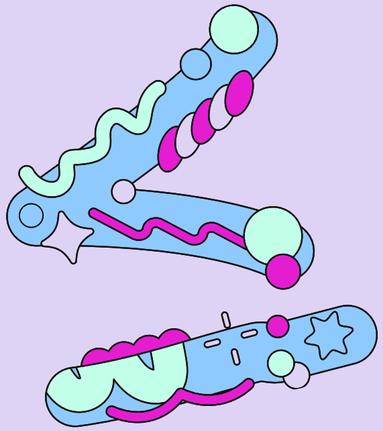
What is x?

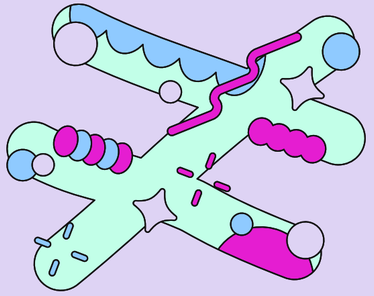
Safety Slide



Answer

97





Direct to team 2...

$f(x)$	$f^{-1}(x)$
	

Question 2

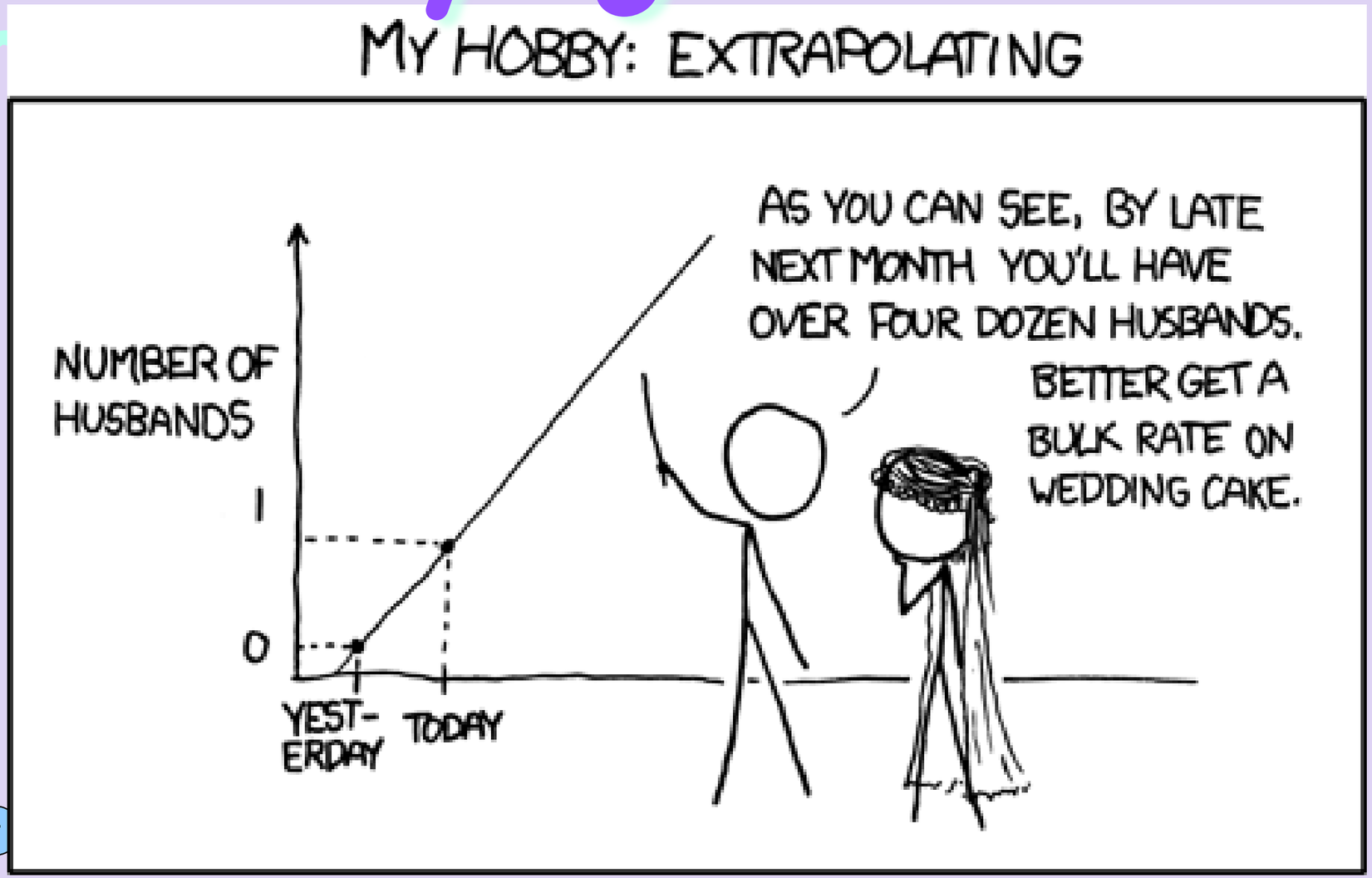
Consider a mapping M from $P \to P$ where P is the set of points on the parabola $S \equiv (y^2 = 4x)$. Let $f(P)$ denote the angle between the tangent at point P and the line joining P to the focus F.

Now we define M as follows:

1) From point P draw tangent to the parabola & let it intersect the directrix at X.
2) Draw a line at X parallel to the axis and let it intersect the parabola at Q.
3) $M(P) = Q$

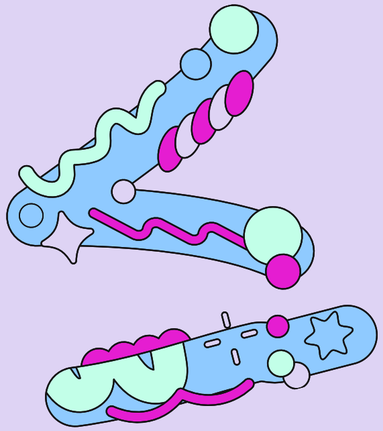
Given that $f(P) = 100^\circ$.
Find $f(M^7(P))$

Safety Slide



Answer

20





Direct to team 3...

Question 3

Let $a_1 = 67$,

$$a_2 = 1 + a_1,$$
$$a_3 = 1 + a_1 a_2,$$
$$\vdots$$
$$a_n = 1 + a_1 a_2 \cdots a_{n-1}.$$

Find the value of

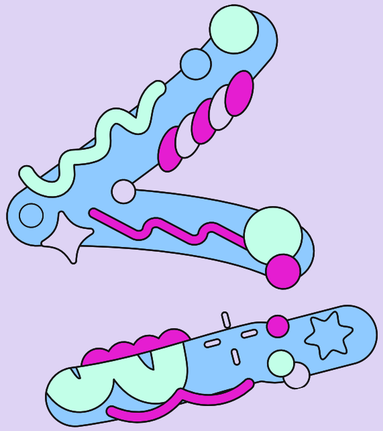
$$\lim_{n \to \infty} \left(\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n} \right).$$

Safety Slide

$$\begin{bmatrix} \cos 90^\circ & \sin 90^\circ \\ -\sin 90^\circ & \cos 90^\circ \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} a_2 \\ -a_1 \end{bmatrix}$$

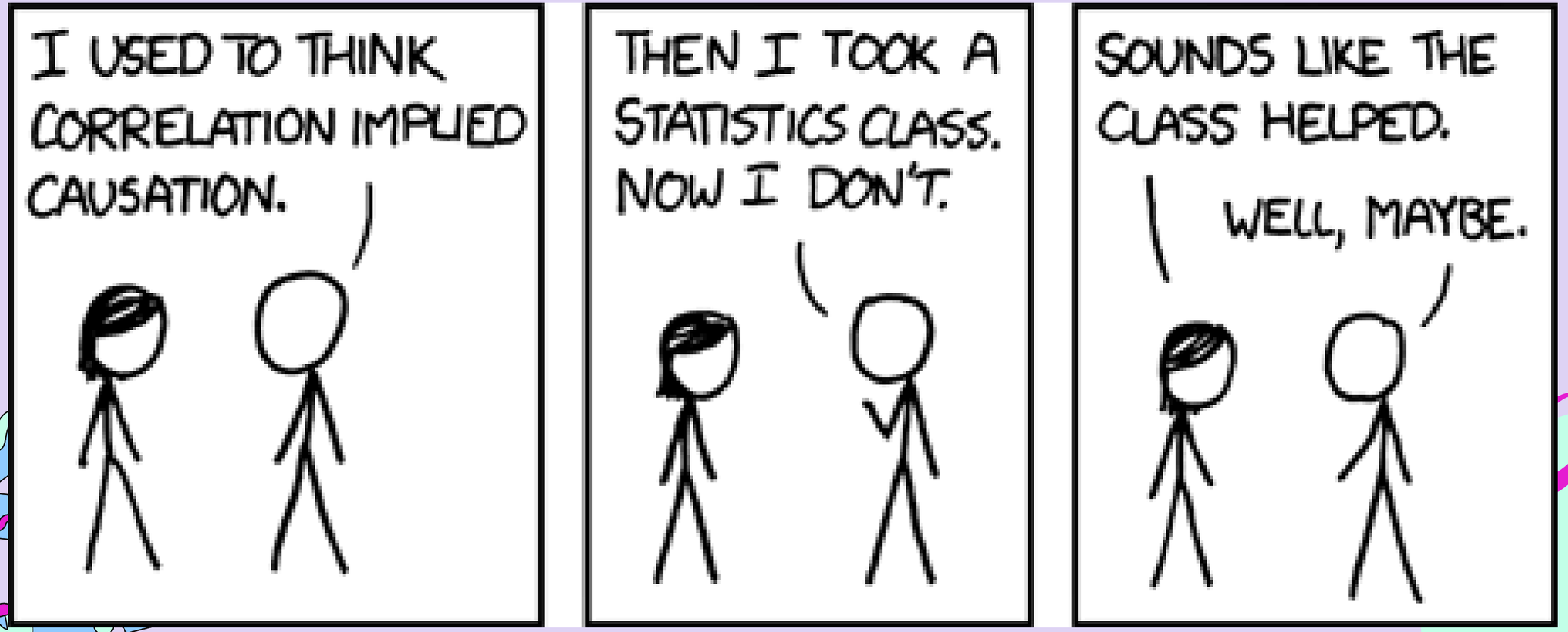
Answer

2/67




6
7

Direct to team 4...



Question 4

Find all functions $f : \mathbb{N} \to \mathbb{N}$ such that for every positive integer n the following is valid:
If $d_1, d_2, \ldots, d_s$ are all the positive divisors of n, then

$$f(d_1)f(d_2)\ldots f(d_s) = n.$$

Find the sum: $f(1) + f(2) + \cdots + f(10)$.

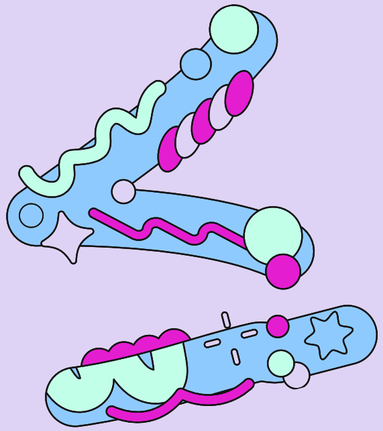
Safety Slide

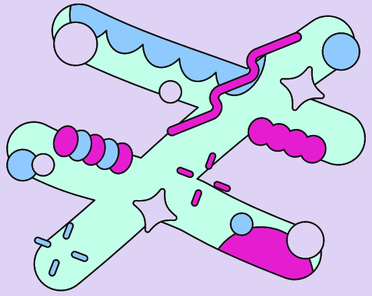
$$\int \text{aspiri}\, dn =$$



Answer

27





Direct to team 5...



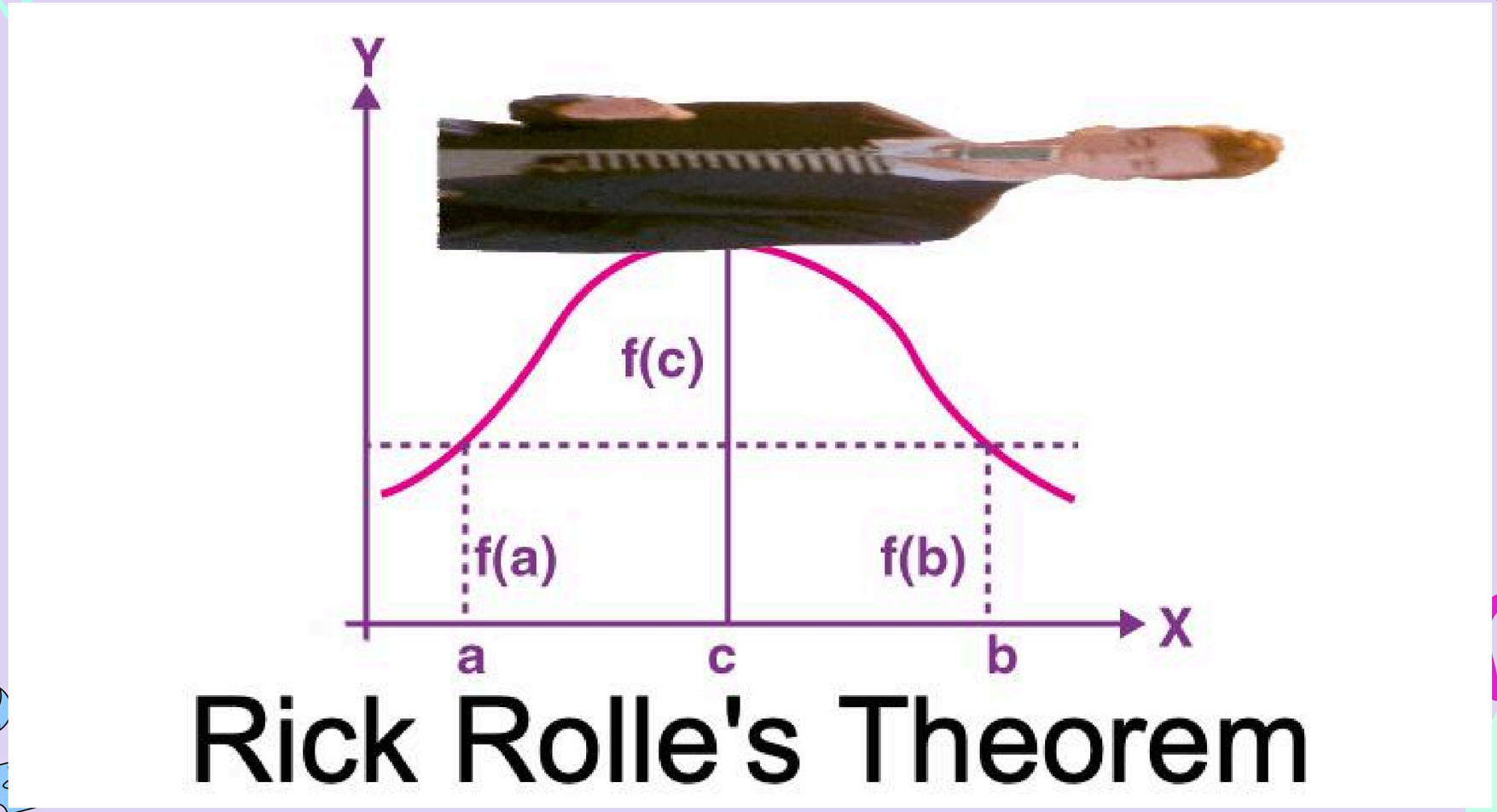
Question 5

Let (x_n) be a sequence of positive real numbers initialized with $x_1 = 1$.
The subsequent terms are generated using the recurrence

$$x_{n+1} = \frac{x_n}{1 + x_n \ln\left(\frac{(n+1)^2}{(n+1)^2 - 1}\right)}.$$

Find $\lim_{n \to \infty} x_n$.

Safety Slide



Answer

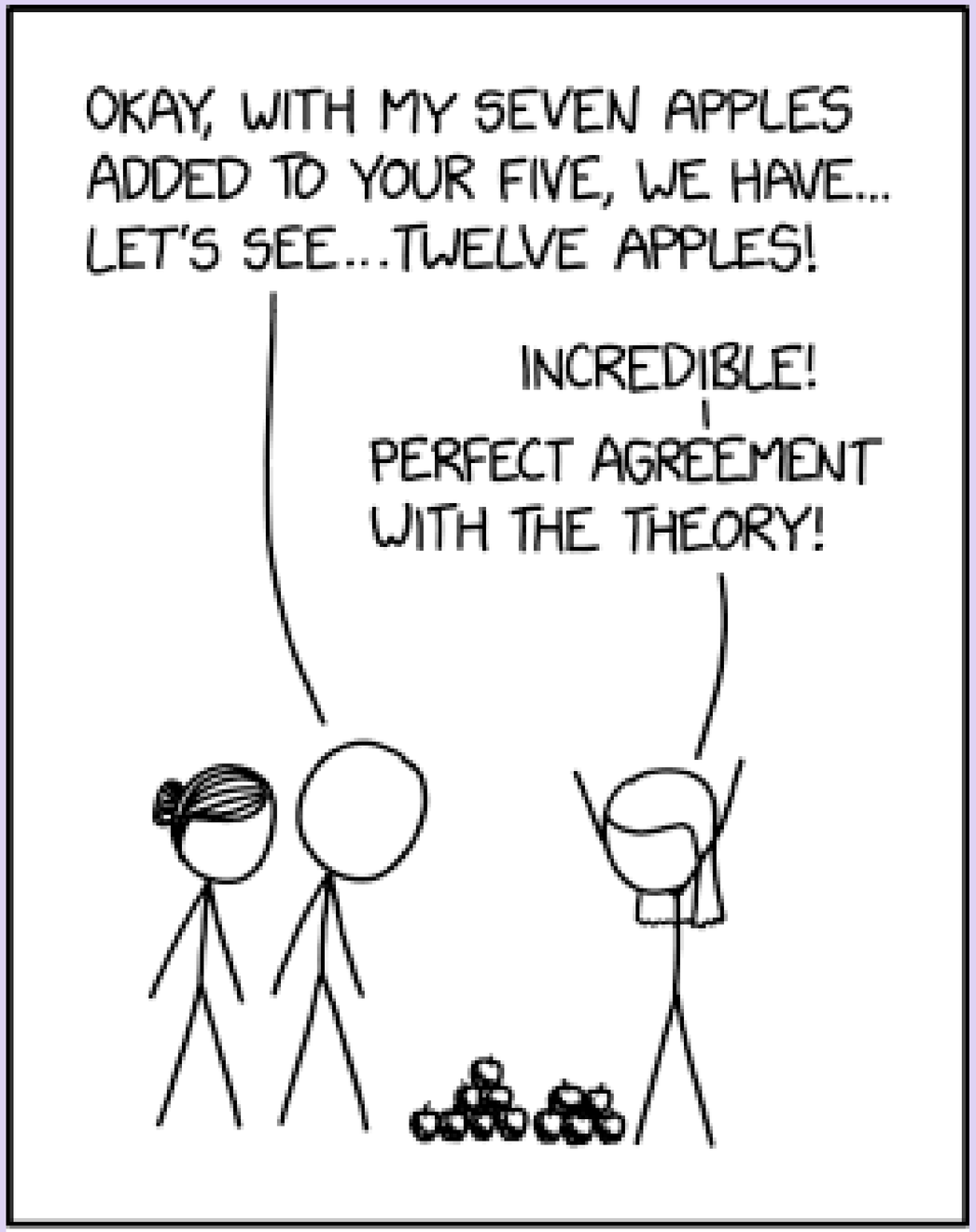
$$1/(1+\ln 2)$$

Direct to team 6...



Question 6

Visharad has a matrix with $m = 4$ rows and $n = 6$ columns.
A matrix M is called valid if and only if
every submatrix of dimensions 2 rows and 2 columns
has an even number of ones in it.
The elements of the matrix are either 0 or 1.
How many possible valid matrices are there?

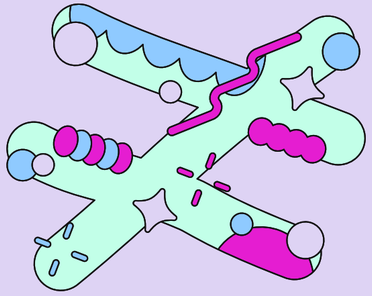


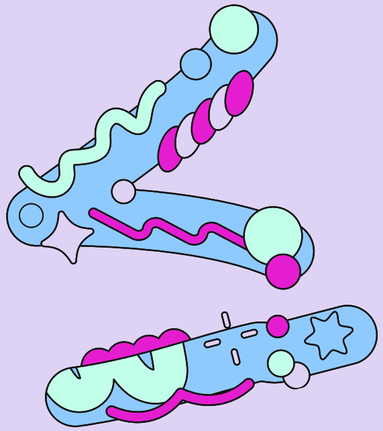
EXPERIMENTAL MATHEMATICIANS

Safety Slide

Answer

512

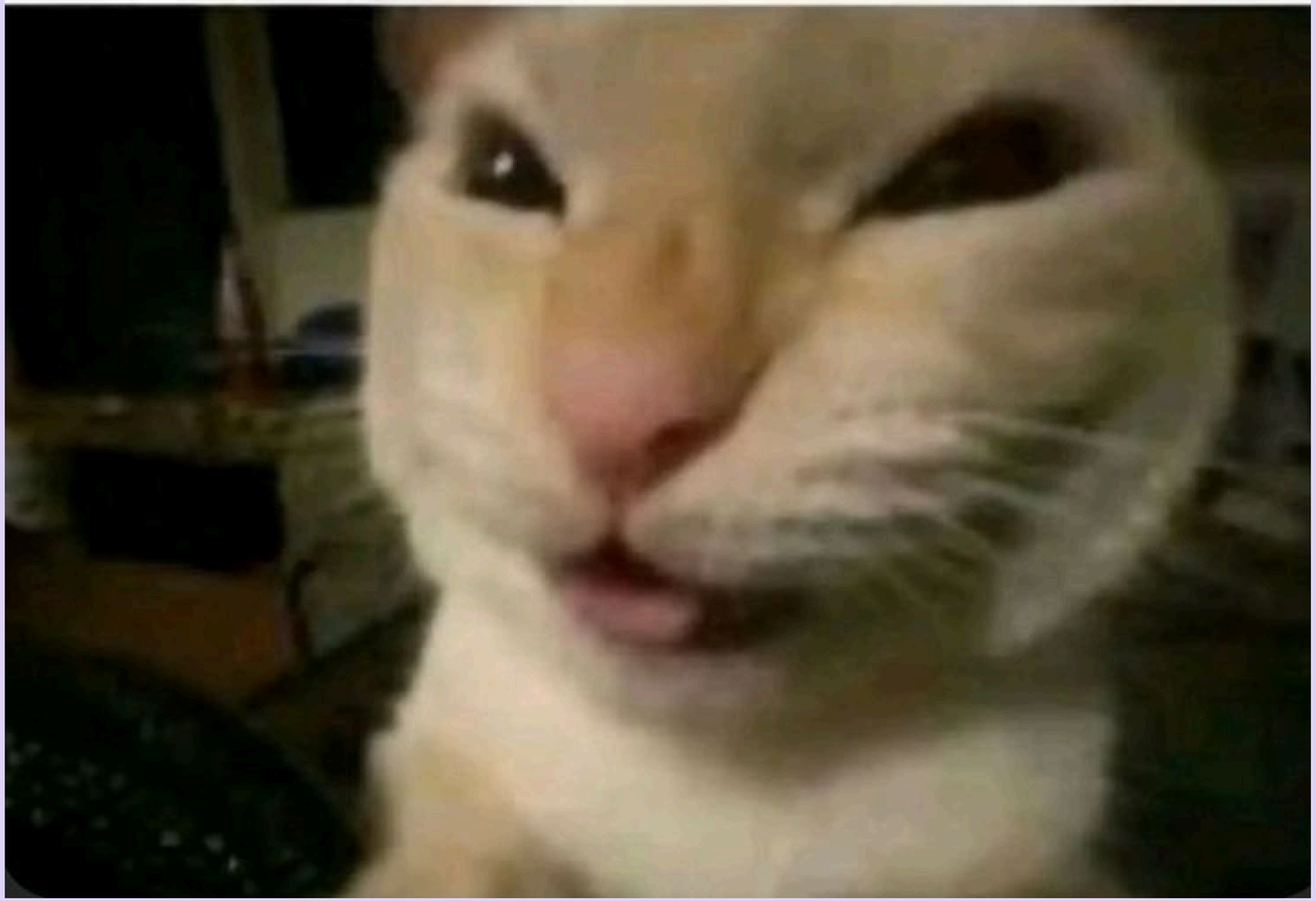




And we are done

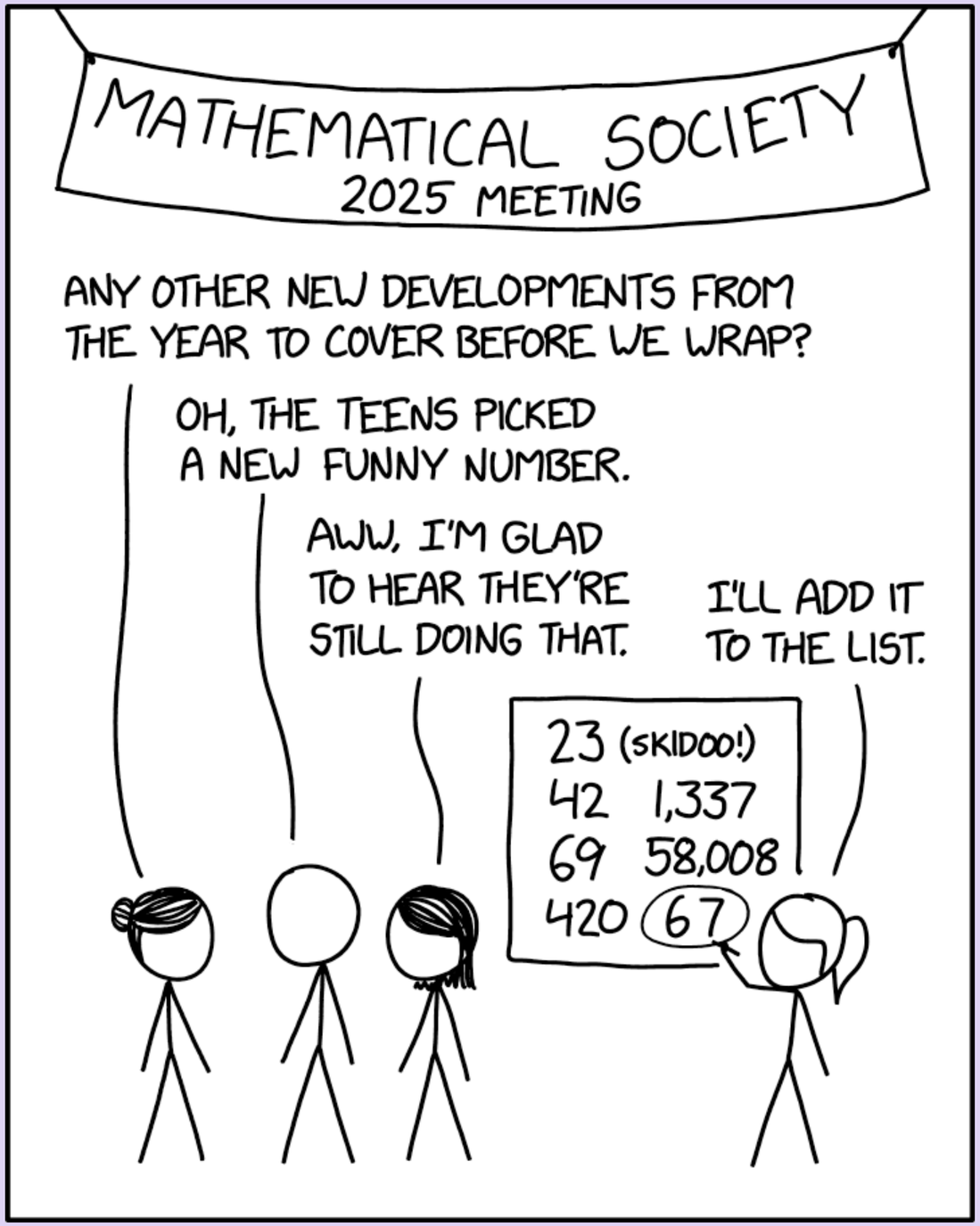


NYEHEHHEJEHEJEHEHE



Question 7

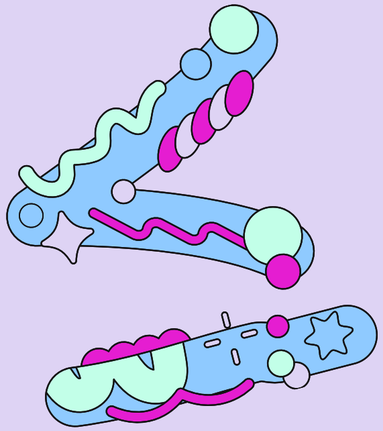
A deck contains N distinct cards numbered 1 to N.
You draw cards one by one without replacement. A “match” occurs at step i if the i-th card drawn has the number i.
However, the deck is rigged: card number 1 is missing and is replaced with a duplicate of card number 2.
For $N = 7,$ determine the exact probability of getting 0 matches in a full deck clearance. If the probability is $\frac{p}{q},$ where p, q are coprime positive integers, find $p + q$.



Safety Slide

Answer

685



Direct to team 2...



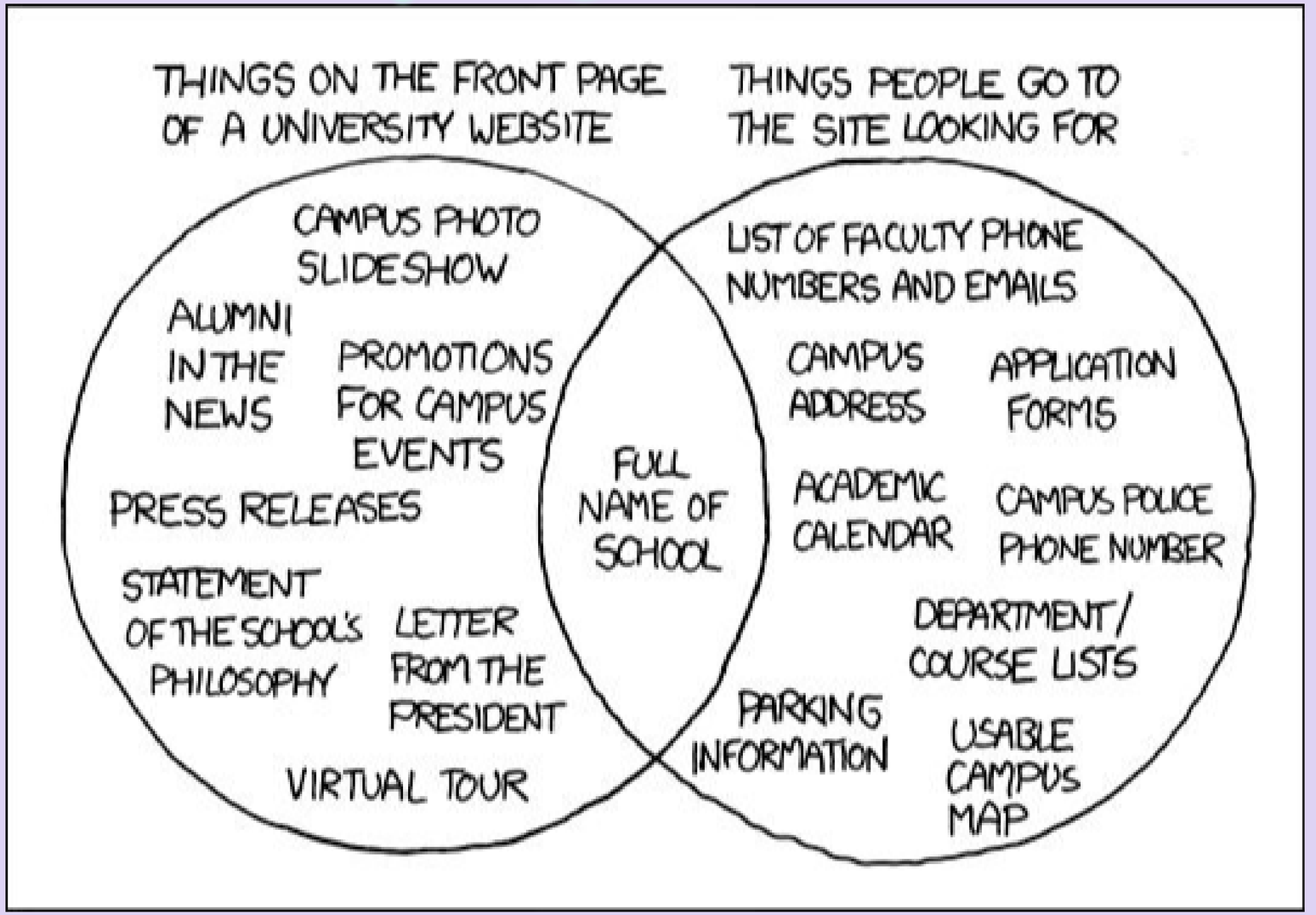
Question 8

Let (x_n) be a sequence of positive real numbers initialized with $x_1 = 1$.
The subsequent terms are generated using the recurrence

$$x_{n+1} = \frac{x_n}{1 + x_n \ln\left(\frac{(n+1)^2}{(n+1)^2 - 1}\right)}.$$

Find $\lim_{n \to \infty} x_n$.

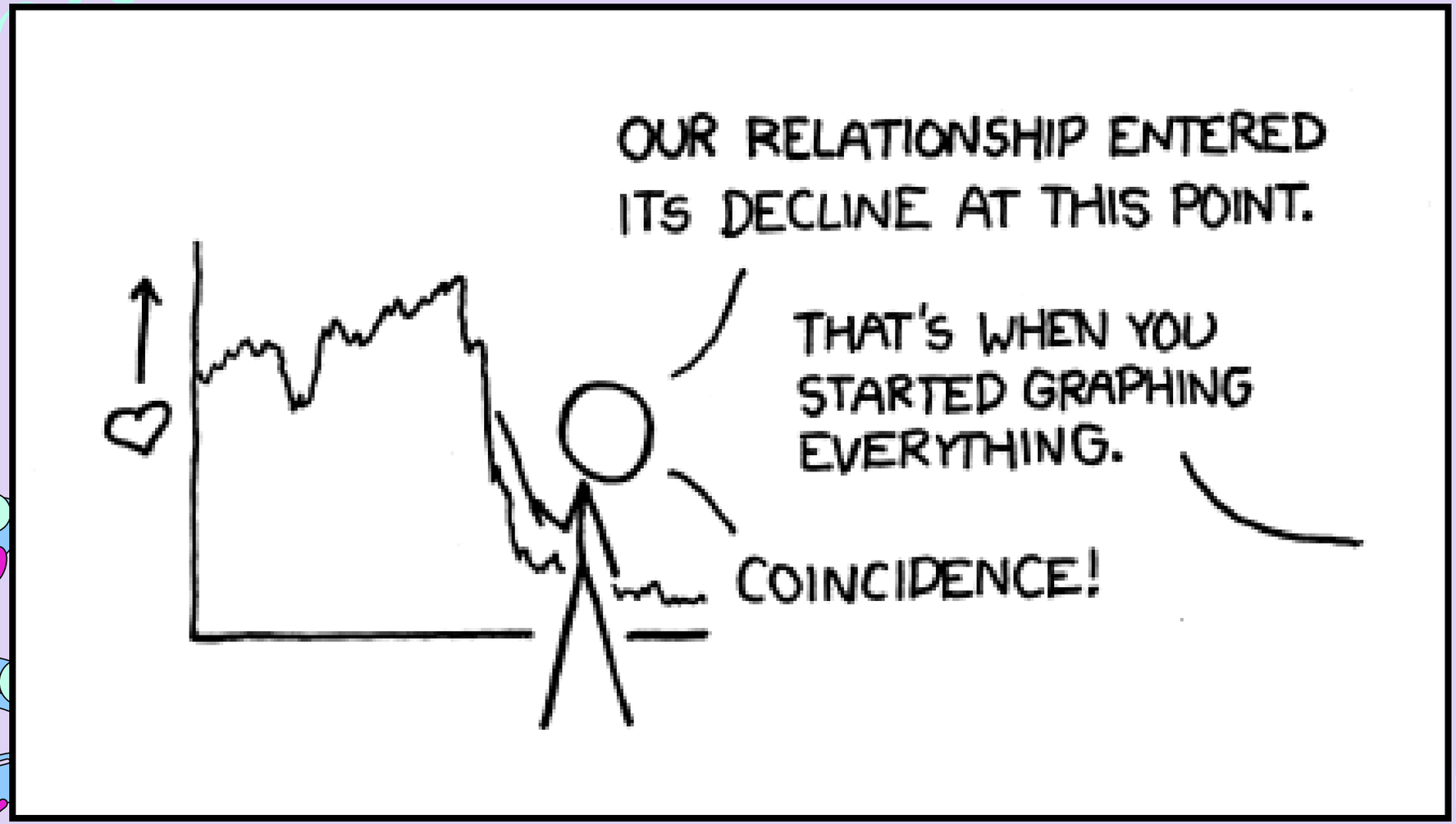
Safety Slide



Answer

$$1/(1+\ln 2)$$

Direct to team 3...



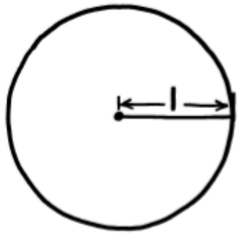
Question 9

Find all functions $f : \mathbb{N} \to \mathbb{N}$ such that for every positive integer n the following is valid:
If $d_1, d_2, \ldots, d_s$ are all the positive divisors of n, then

$$f(d_1)f(d_2)\ldots f(d_s) = n.$$

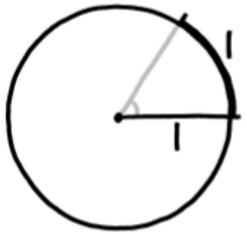
Find the sum: $f(1) + f(2) + \cdots + f(10)$.

MATH FACTS



THE UNIT CIRCLE HAS A RADIUS OF ONE

✓ NORMAL



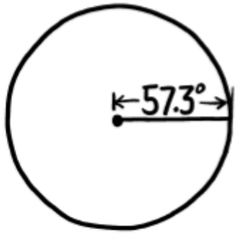
ONE RADIAN EQUALS THE LENGTH OF A CIRCLE'S RADIUS

✓ NORMAL



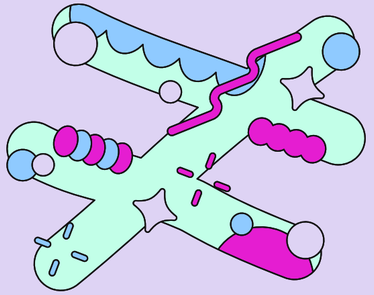
ONE RADIAN IS 57.3 DEGREES

✓ NORMAL

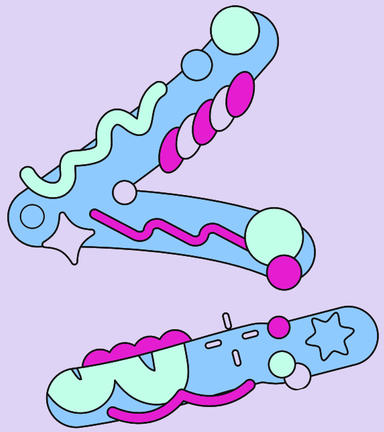


THE UNIT CIRCLE HAS A RADIUS OF 57.3 DEGREES

✗ CURSED

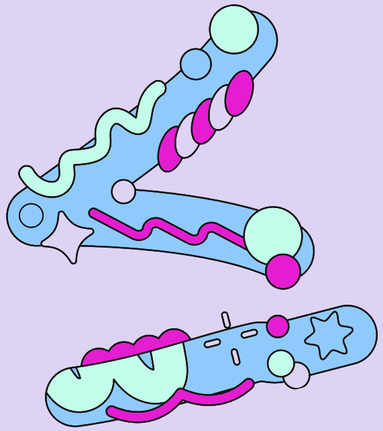


Safety Slide



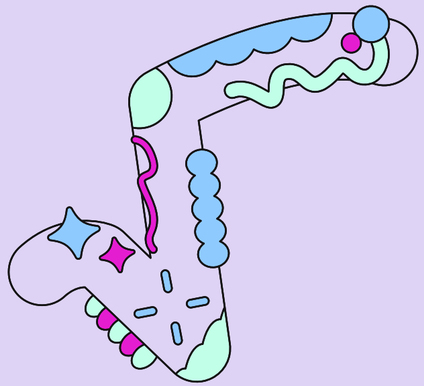
Answer

27

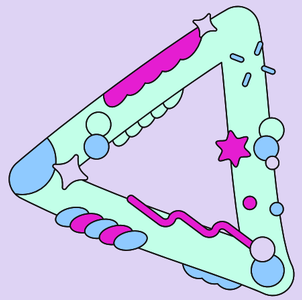


Direct to team 4...

You're given a complicated math problem	
Things start to cancel out	
Your answer is a whole number	
Your answer is one of the options	



Question 10



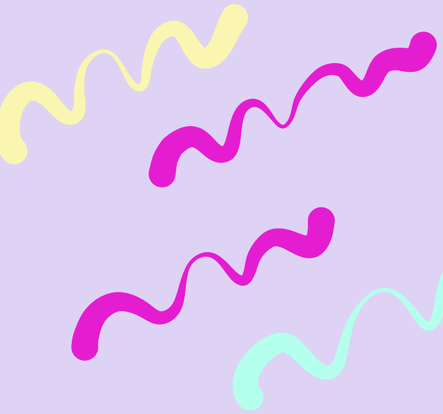
Let $a_1 = 67$,

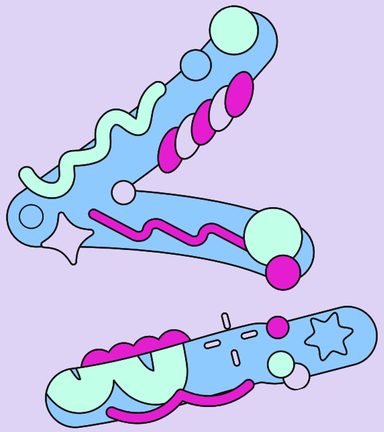
$$a_2 = 1 + a_1,$$
$$a_3 = 1 + a_1 a_2,$$
$$\vdots$$
$$a_n = 1 + a_1 a_2 \cdots a_{n-1}.$$

Find the value of

$$\lim_{n \to \infty} \left(\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n} \right).$$







A GUIDE TO

INTEGRATION BY PARTS:

GIVEN A PROBLEM OF THE FORM:

$$\int f(x)\,g(x)\,dx = ?$$

CHOOSE VARIABLES u AND v SUCH THAT:

$$u = f(x)$$
$$dv = g(x)\,dx$$

NOW THE ORIGINAL EXPRESSION BECOMES:

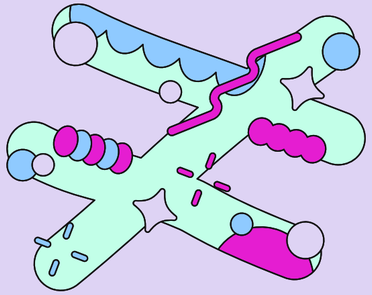
$$\int u\,dv = ?$$

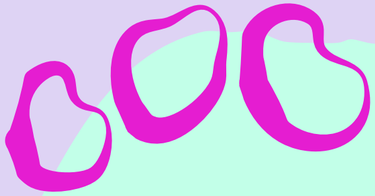
WHICH *DEFINITELY* LOOKS EASIER.

ANYWAY, I GOTTA RUN.

BUT GOOD LUCK!

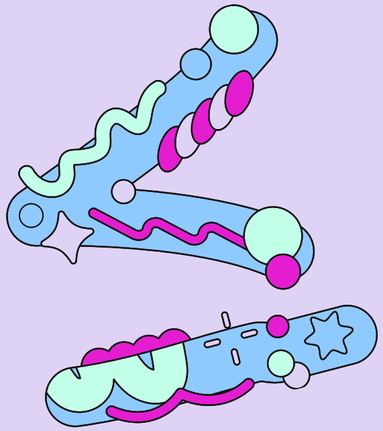
Safety Slide





Answer

2/67





Direct to team 5...

Question 11

Consider a mapping M from $P \to P$ where P is the set of points on the parabola $S \equiv (y^2 = 4x)$. Let $f(P)$ denote the angle between the tangent at point P and the line joining P to the focus F.

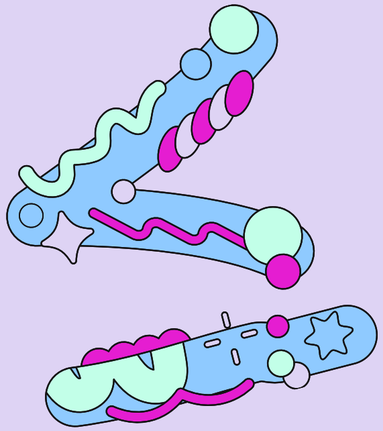
Now we define M as follows:

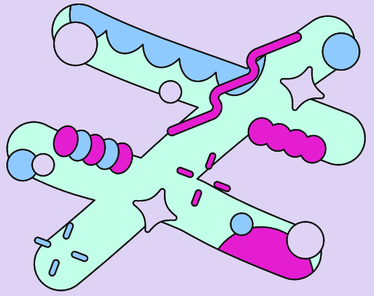
1) From point P draw tangent to the parabola & let it intersect the directrix at X.
2) Draw a line at X parallel to the axis and let it intersect the parabola at Q.
3) $M(P) = Q$

Given that $f(P) = 100^\circ$.
Find $f(M^7(P))$

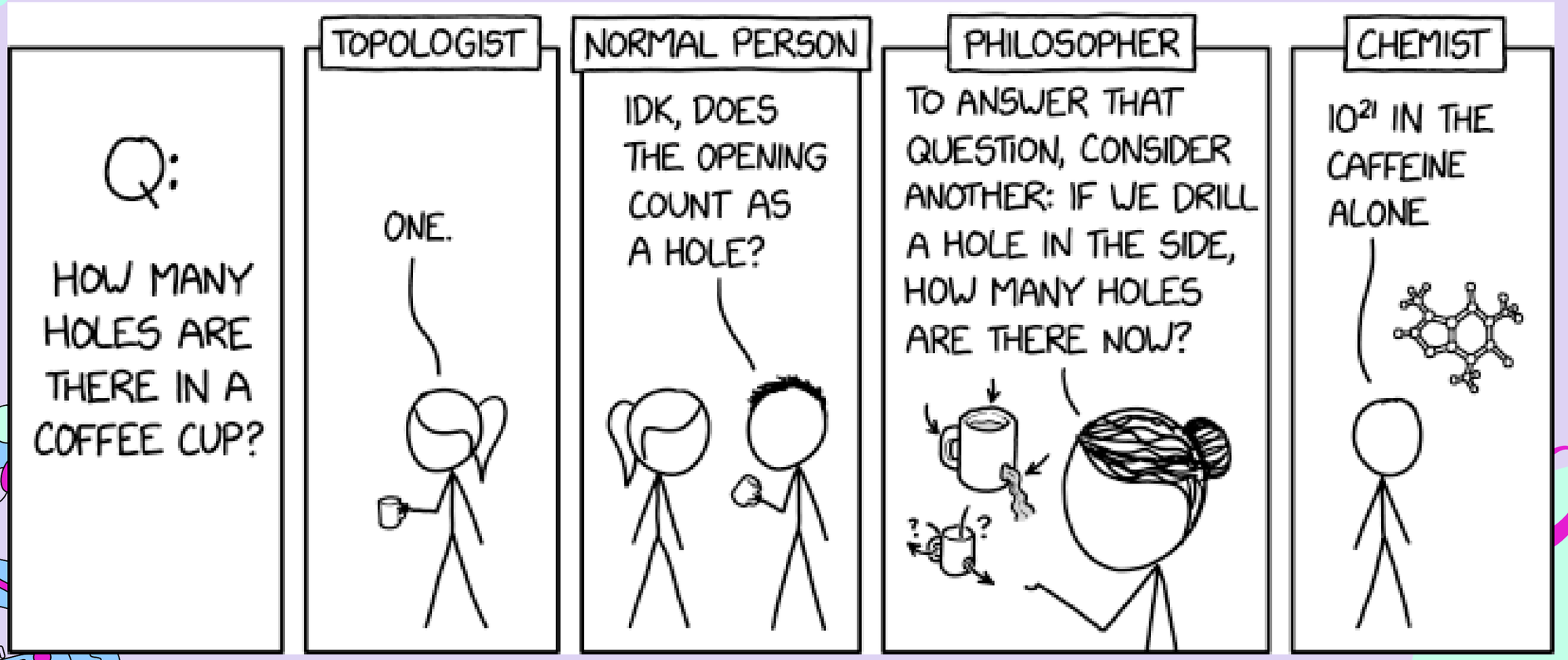
Answer

20





Direct to team 6...



Question 12

Visharad has a degree 5 polynomial P which satisfies

$$P(t) = 2^t \qquad \text{for } t = 0, 1, 2, 3, 4, 5.$$

Not knowing that P is a polynomial, he assumes

$$P(t) = 2^t \qquad \text{for all } t.$$

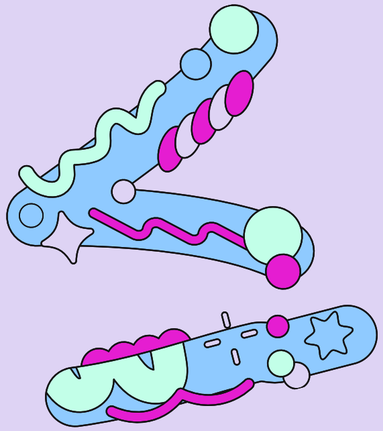
He uses this to calculate $P(6) + P(7)$.
Find by how much Visharad's answer is off.

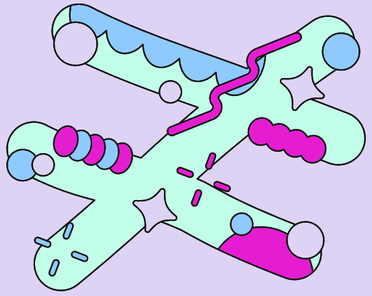
Safety Slide



Answer

9






Now it's time
for da
Buzzerrrrrrr