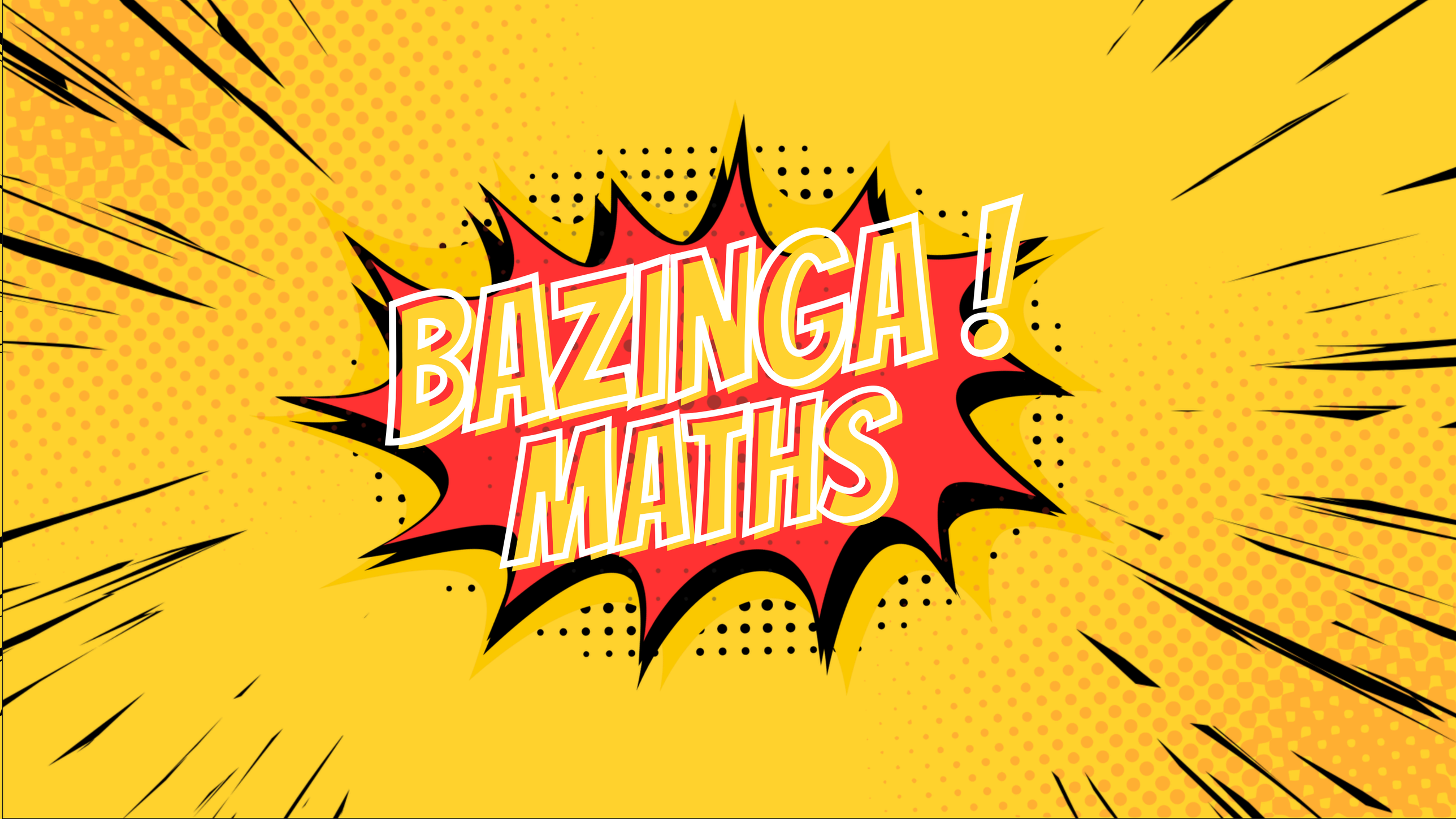

BAZINGA!
MATHS


PRELIMS

IMPORTANT

DIRECTIONS & GUIDELINES



One by one, **15** questions will be shown on the screen for **90** seconds each.



After one run-through of all questions, they will be shown again in the same order, for **30** seconds each.



Write all answers neatly in the sheet provided (withyour name and roll number on top).



For multiple choice questions, just write the option, like Q1. a).



In the event of a tie, the person who solves more **star**-marked questions will qualify.

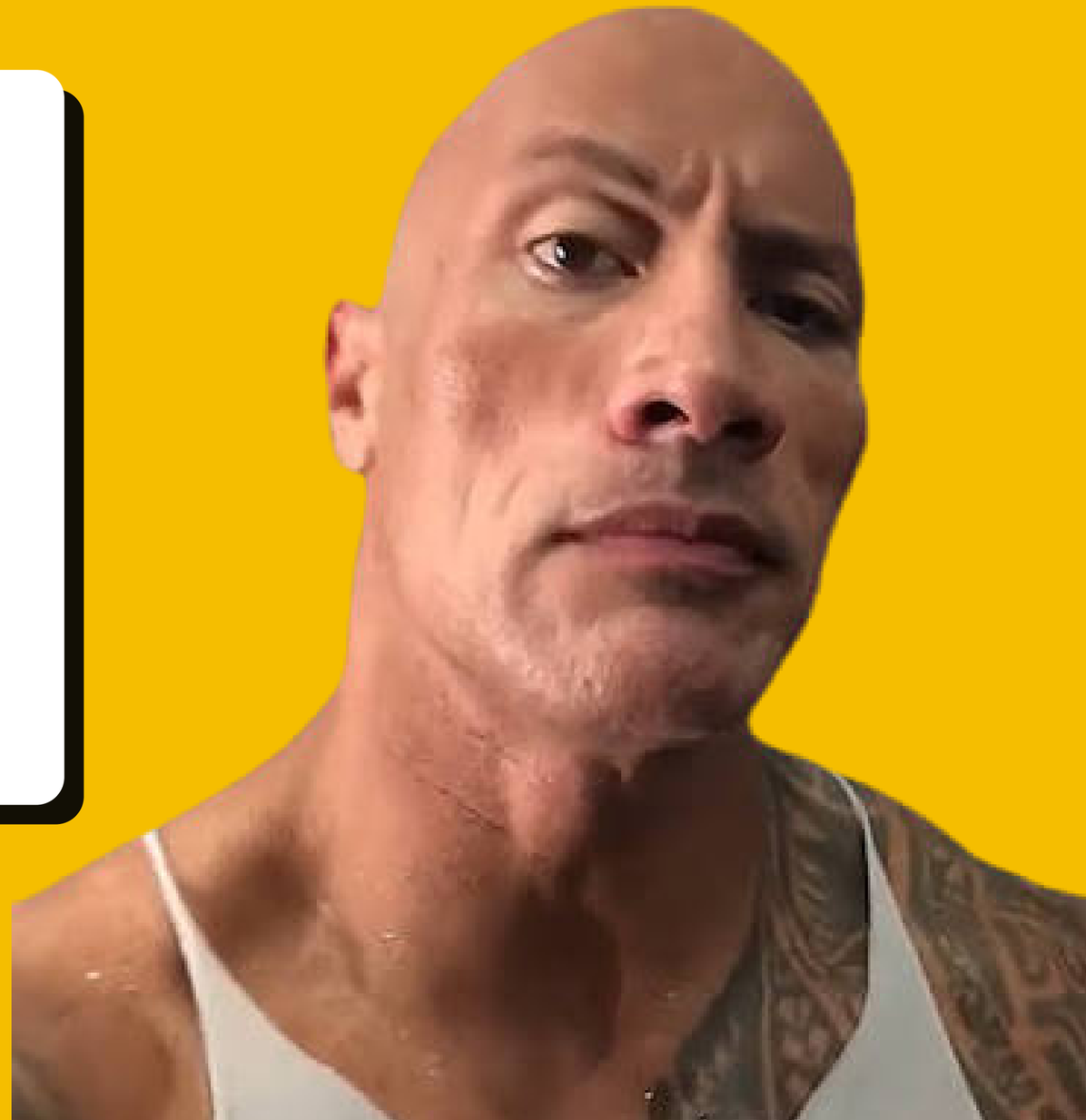
⚠ INCOMING REALITY CHECK

"15 QUESTIONS?"

"90 SECONDS?"

" PIECE OF CAKE "

FAMOUS LAST WORDS 💀



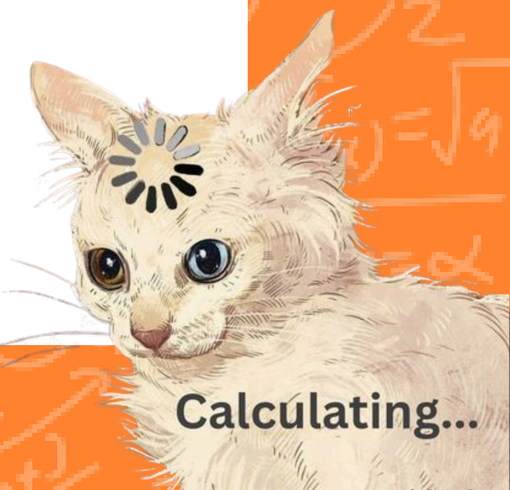
SAFETY SLIDE

QUESTION 1 COMING UP...

PENS READY • ANSWER SHEETS READY • ALL THE BEST!

QUESTION 1

Find the number of sets in the power set of $\{1, 3, 5, 7, \ldots, 89\}$ whose sum of elements is less than or equal to 1012.

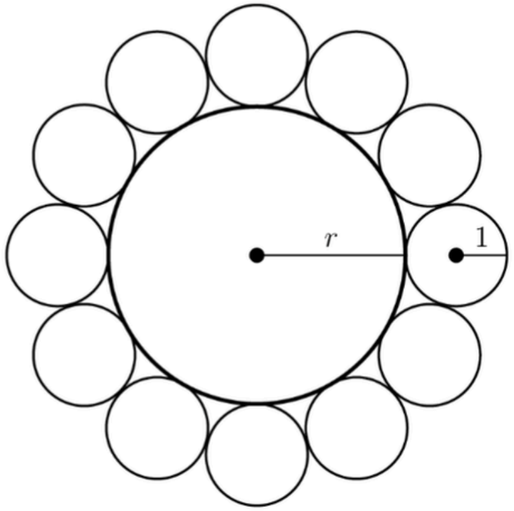


SAFETY SLIDE

QUESTION 2 COMING UP...

QUESTION 2

A circle of radius r is surrounded by 12 circles of radius 1 externally tangent to the central circle and sequentially tangent to each other, as shown. Then r can be written as $\sqrt{a}+\sqrt{b}+c,$ where a, b, c are integers. What is $a+b+c$?





SAFETY SLIDE



Trust my calculation

Trust my first instinct

QUESTION 3 COMING UP...

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Suppose that $a, b, c,$ and d are positive integers satisfying all of the following relations:

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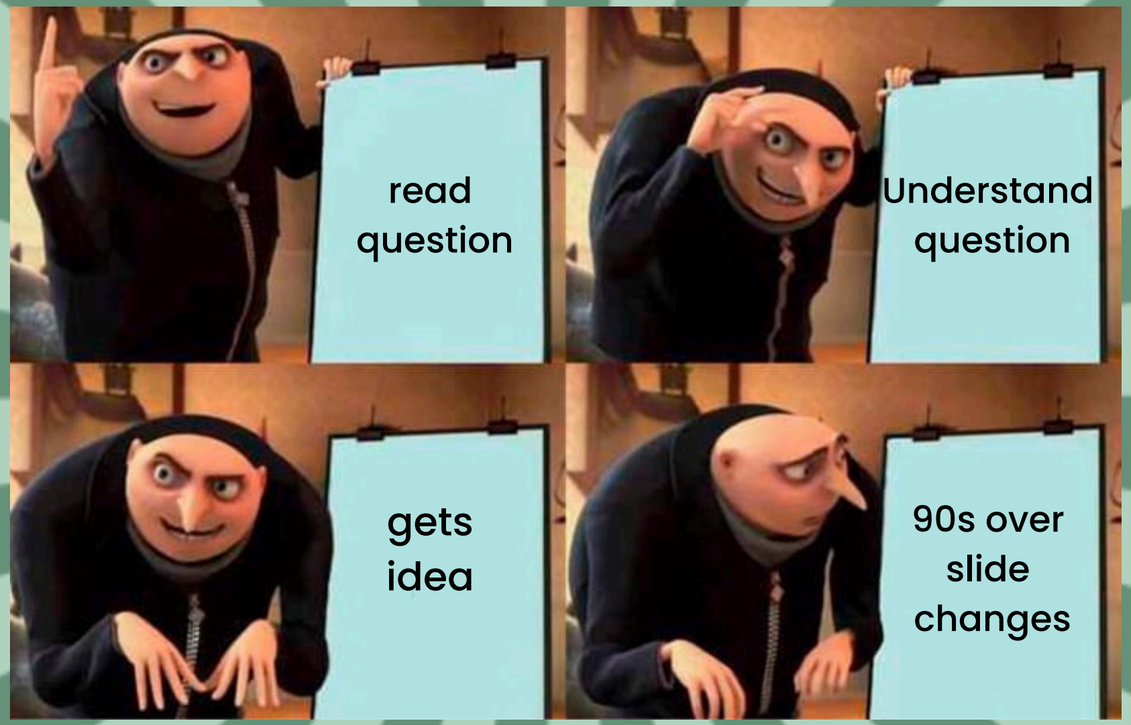
$$\text{lcm}(b, d) = 2^2 \cdot 3^3 \cdot 5^2$$

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What is $\gcd(a, b, c, d)$?



SAFETY SLIDE



QUESTION 4 COMING UP...

QUESTION 4

$$\text{If } \lim_{n\to\infty} a^n \left(2 - \underbrace{\sqrt{2+\sqrt{2+\ldots}}}_{n \text{ terms}} \right) = b, \text{ find } a \cdot b \ (\neq 0).$$

(By n terms it means there are n 2s in the expression.)



SAFETY SLIDE

QUESTION 5 COMING UP...

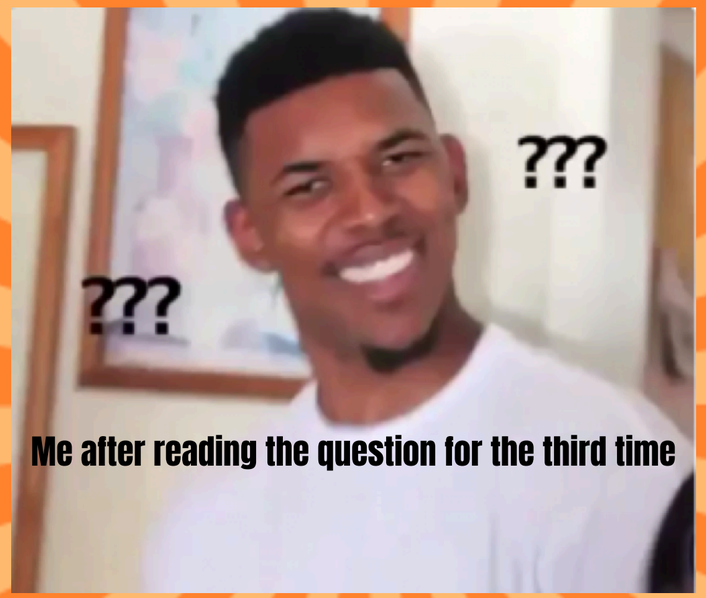
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X is a class of numbers with a very simple definition, but has been widely studied by mathematicians as great as Leonhard Euler. The latter proved a long conjectured statement, that the formula $2^{p-1}(2^p - 1)$ in fact produces all even numbers of this class. Yet the X numbers are shrouded in mystery, as no one knows if any odd numbers of this class exist, nor whether this class is infinite.

Identify X.



SAFETY SLIDE



QUESTION 6 COMING UP...

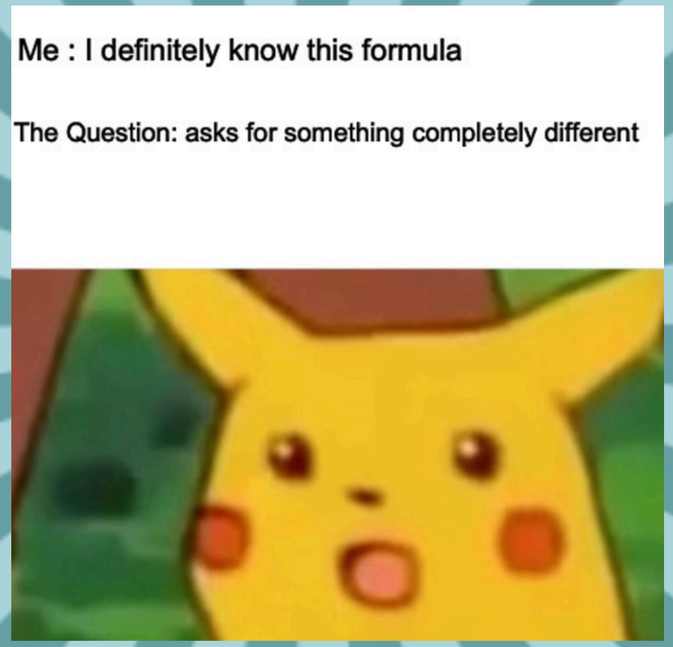
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Find all pairs $(x, y) \in \mathbb{Z}^2$ such that

$$x^3 + x^2y + xy^2 + y^3 = 8(x^2 + xy + y^2 + 1).$$



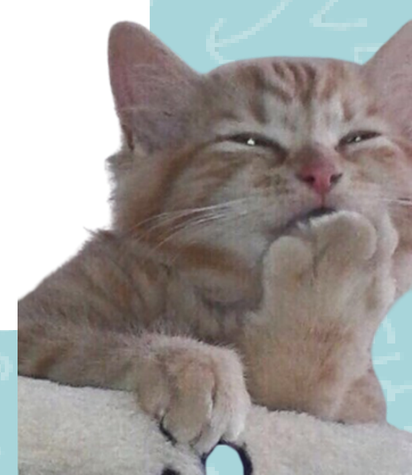
SAFETY SLIDE



QUESTION 7 COMING UP...

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SAFETY SLIDE

QUESTION 8 COMING UP...

QUESTION 8

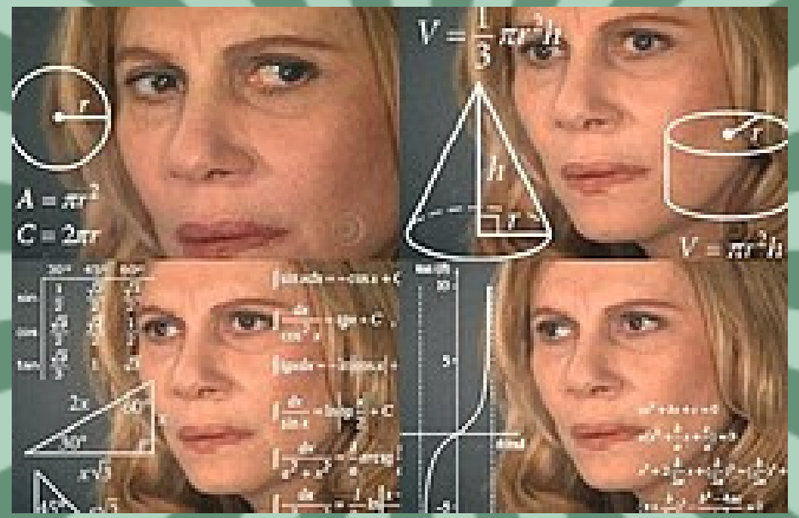
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SAFETY SLIDE



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At a competition with N players, the number of players given elite status is equal to $2^{1+\lfloor \log_2(N-1) \rfloor} - N$.

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SAFETY SLIDE

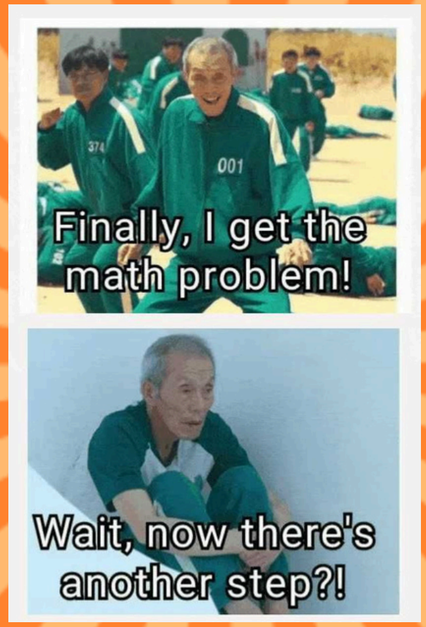
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SAFETY SLIDE



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SAFETY SLIDE



QUESTION 12 COMING UP...

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SAFETY SLIDE

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QUESTION 14 COMING UP...

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Find the maximum number of elements S can have.



SAFETY SLIDE

QUESTION 15 COMING UP...

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SECOND LOOK 👀

Same questions. Less mercy.



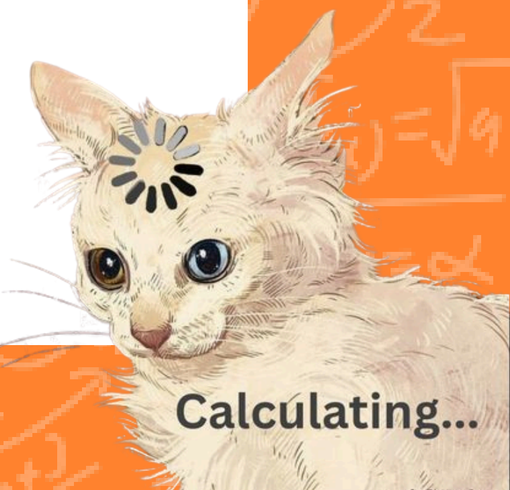
SAFETY SLIDE



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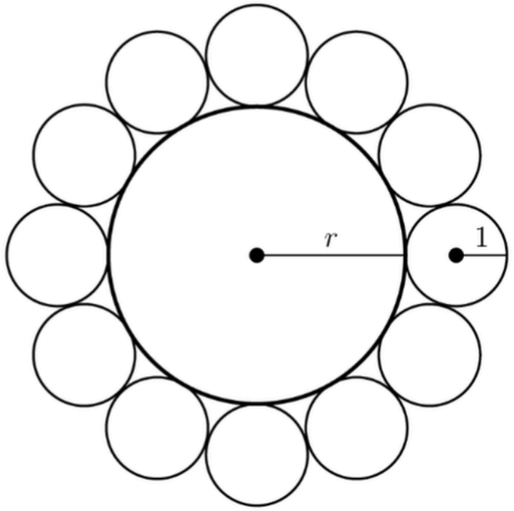


SAFETY SLIDE

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SAFETY SLIDE



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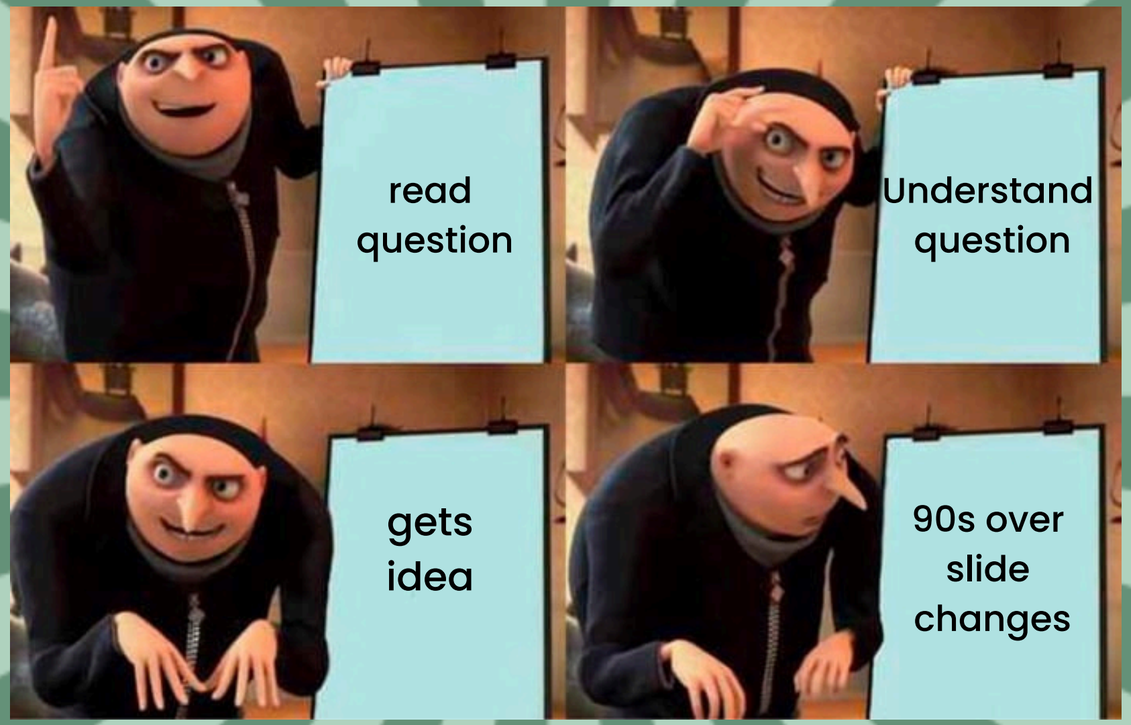
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SAFETY SLIDE



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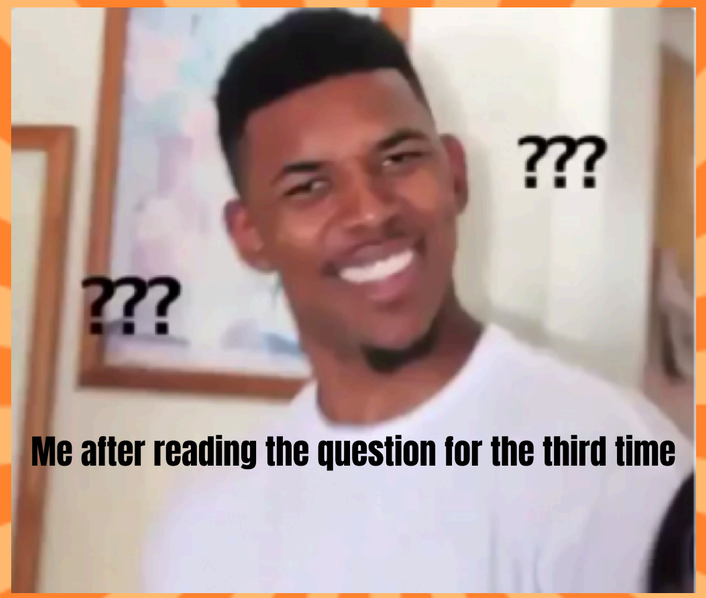
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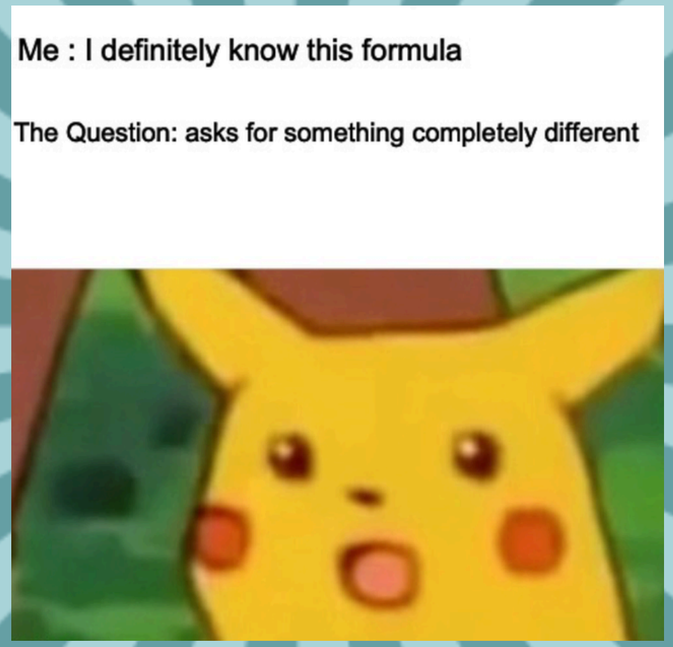
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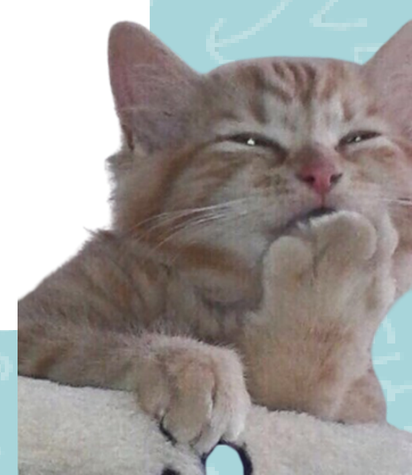
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SAFETY SLIDE

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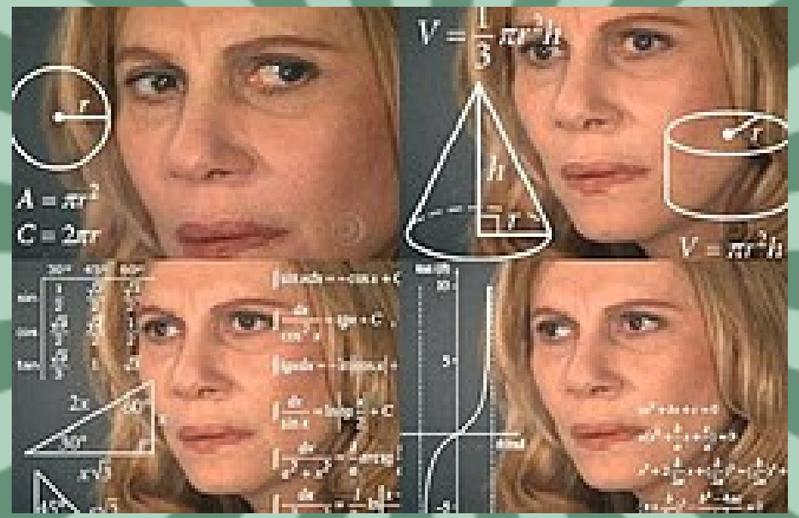
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SAFETY SLIDE

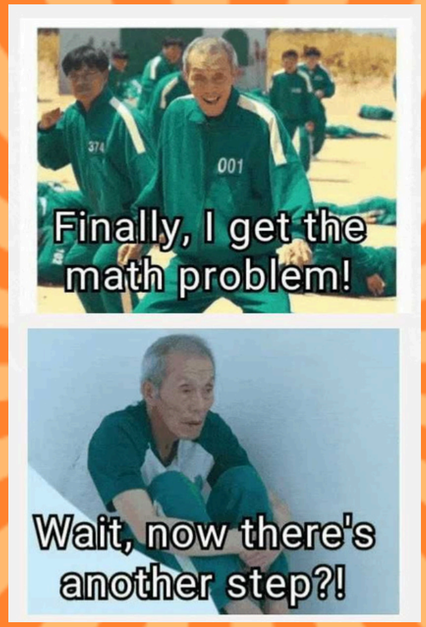
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SAFETY SLIDE



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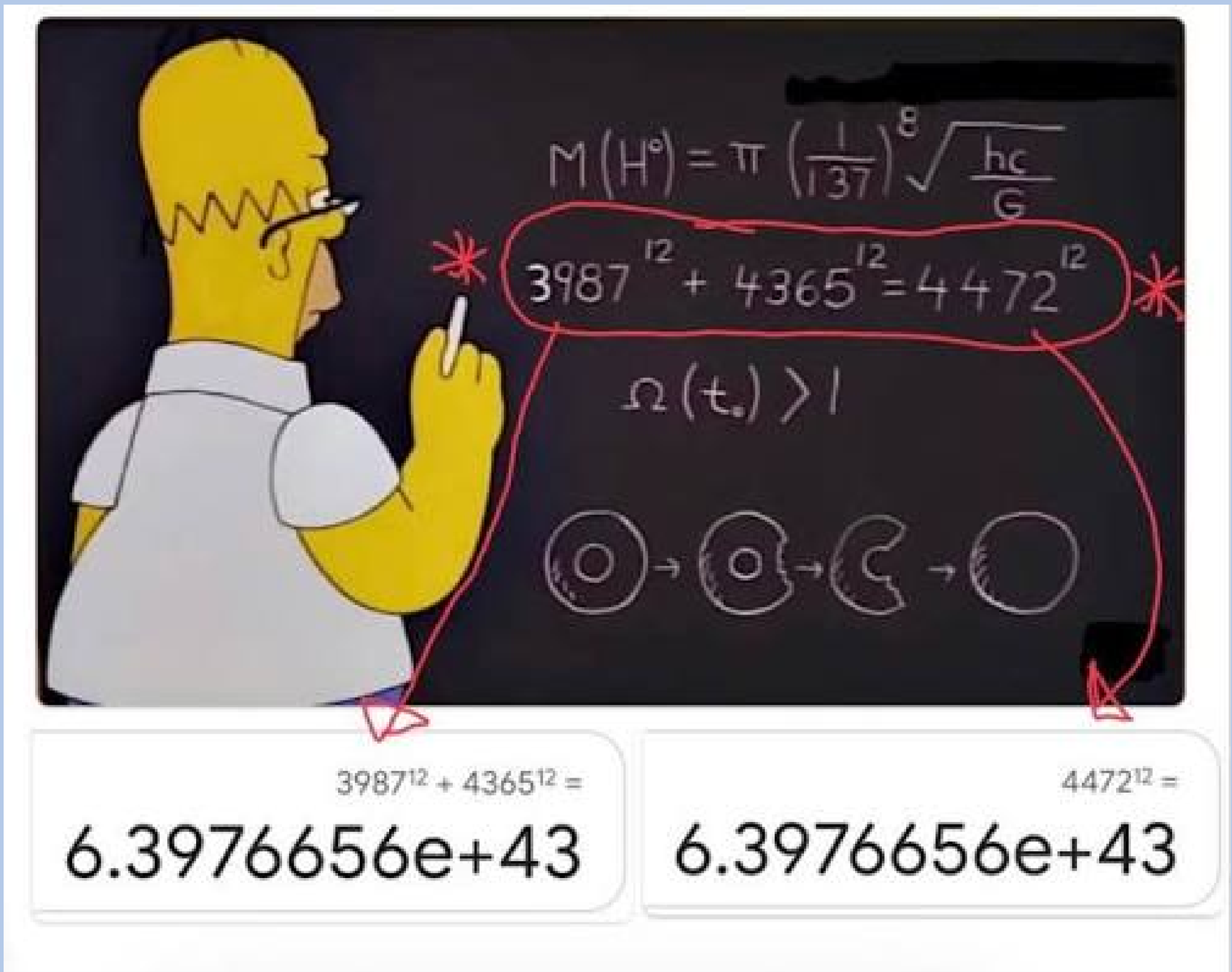
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$$3987^{12} + 4365^{12} =$$

$$4472^{12}$$

SAFETY SLIDE



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SAFETY SLIDE

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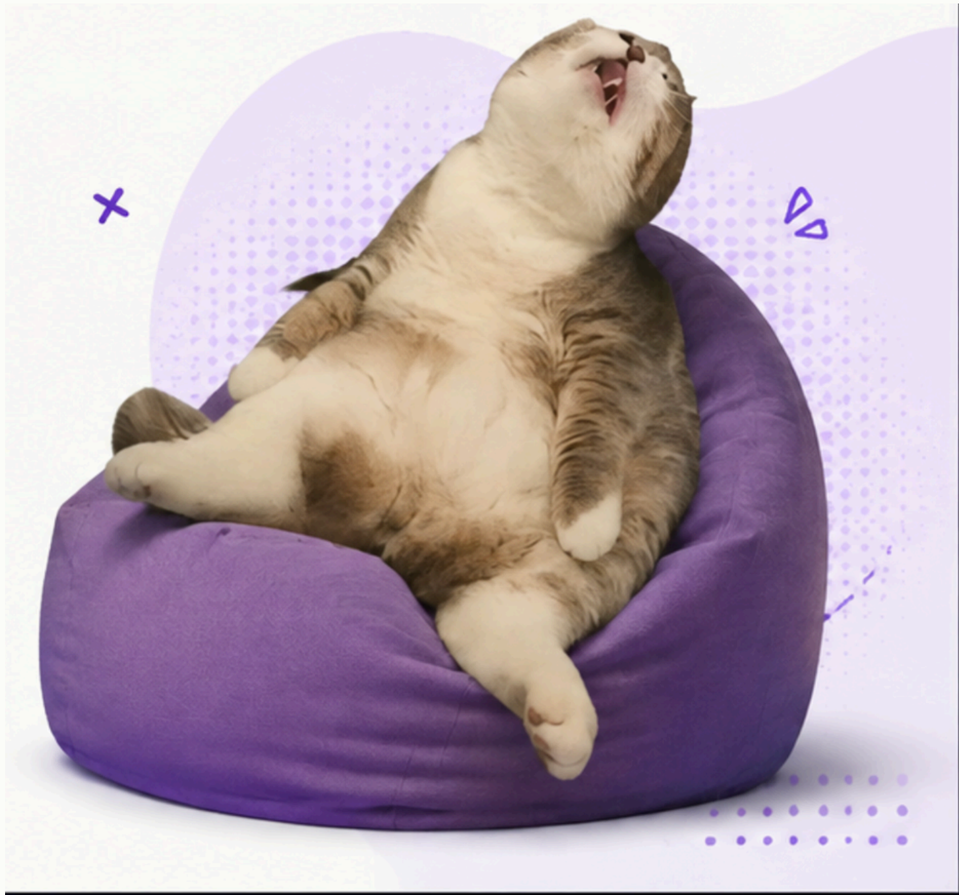
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YOU SURVIVED 🫡

Rest those brain cells

We’ll start cooking again soon. 🔥



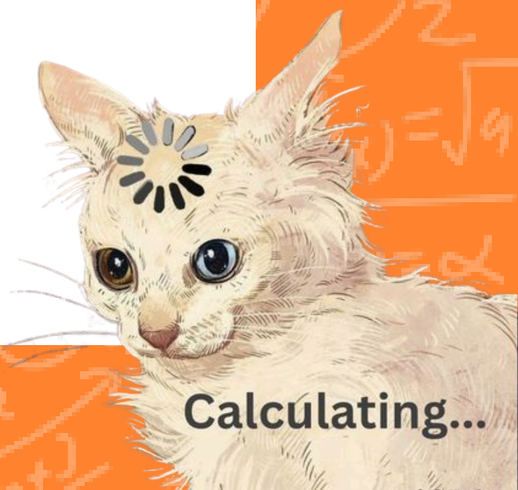
BAZINGA

The answers are about to enter the chat...



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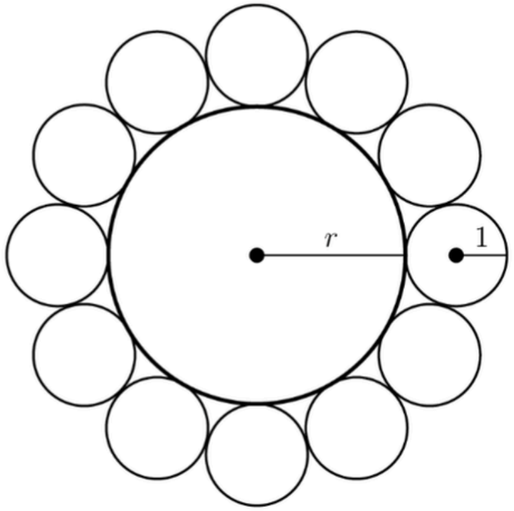


ANSWER

$$2^{44}$$

QUESTION 2

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ANSWER

7

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What is $\gcd(a, b, c, d)$?



ANSWER

3

QUESTION 4

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ANSWER

$$\pi^2$$

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ANSWER

Perfect Numbers

QUESTION 6

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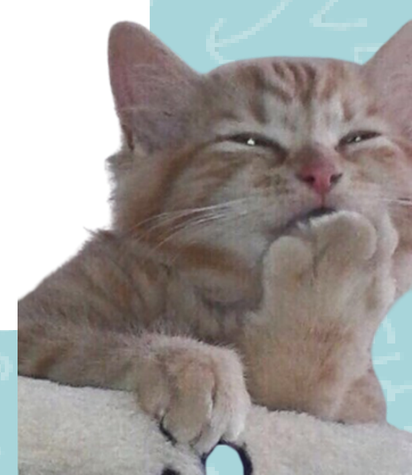


ANSWER

$$(2, 8)\ (8, 2)$$

QUESTION 7

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ANSWER

3

QUESTION 8

Find all pairwise coprime positive integers l, m, n such that

$$(l+m+n)\left(\frac{1}{l}+\frac{1}{m}+\frac{1}{n}\right)$$

is an integer.



ANSWER

$$(1,1,1),(1,1,2),(1,2,3)$$

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ANSWER

154

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ANSWER

840

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ANSWER

Sophie Germain

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ANSWER

48

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ANSWER

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ANSWER

969

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ANSWER

11

THE END