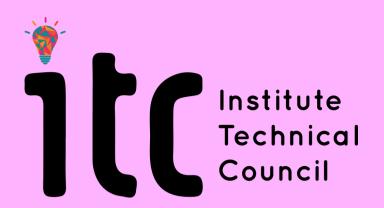


# The Ultimate Maths SHOWDOWN













# Preliminaries

Salar guesam

#### Rules

 This round has 12 questions. We will cycle through them twice: for the first time, we'll be displaying the question on screen for
 90 seconds and the quizmaster will read the question aloud.

• For the second time, the question will not be read aloud and will be shown for **30 seconds** only.

• Please write down the answers to the questions on the sheet of paper given to you in the **correct order**, leaving blanks for any question you have not answered.

• Please also write your name and roll number at the top of the sheet.



Find the largest closed interval over which the function

$$\sqrt{x+2\sqrt{x-1}}+\sqrt{x-2\sqrt{x-1}}$$
 is constant.



After which famous Hungarian mathematician is a "collaborative distance" defined (between himself and any other person), as measured by coauthorships of scientific papers?

He is known for believing in an abstract object called "The Book", in which he claimed God kept the best and most elegant proofs of theorems.



Find the expectation of the minimum of 'n' independent random variables all uniformly distributed in [0, 1].



Determine the number of functions f from the set  $\{1, 2, 3, 4, 5\}$  to itself such that the f(x) = f(f(x)) holds for every x in that set.



Find all triplets of integers (x, y, z) such that

$$xy(x^2-y^2)+yz(y^2-z^2)+zx(z^2-x^2)=1$$



Find all primes p, q such that

$$p^3 + 3q^3 - 32$$

is also prime.



Let n be a natural number. Put

$$x=\left(1+rac{1}{n}
ight)^n$$
 and  $y=\left(1+rac{1}{n}
ight)^{n+1}$ 

Which is bigger:  $x^y$  or  $y^x$ ?



Find the area of the surface  $A \cap B$ , where

$$A=\{(x,y,z)\in\mathbb{R}^3\ |\ x^2+y^2\leq 4, 0\leq z\}$$
 and

$$B = \{(x, y, z) \in \mathbb{R}^3 \mid 0 \le z = y\}$$



Find all pairs of positive integers (n, k) such that

$$n^3 - 2 = k!$$



Determine the value of

$$\int\limits_{0}^{20}(x^{2}-\lfloor x \rfloor \lceil x \rceil)dx$$



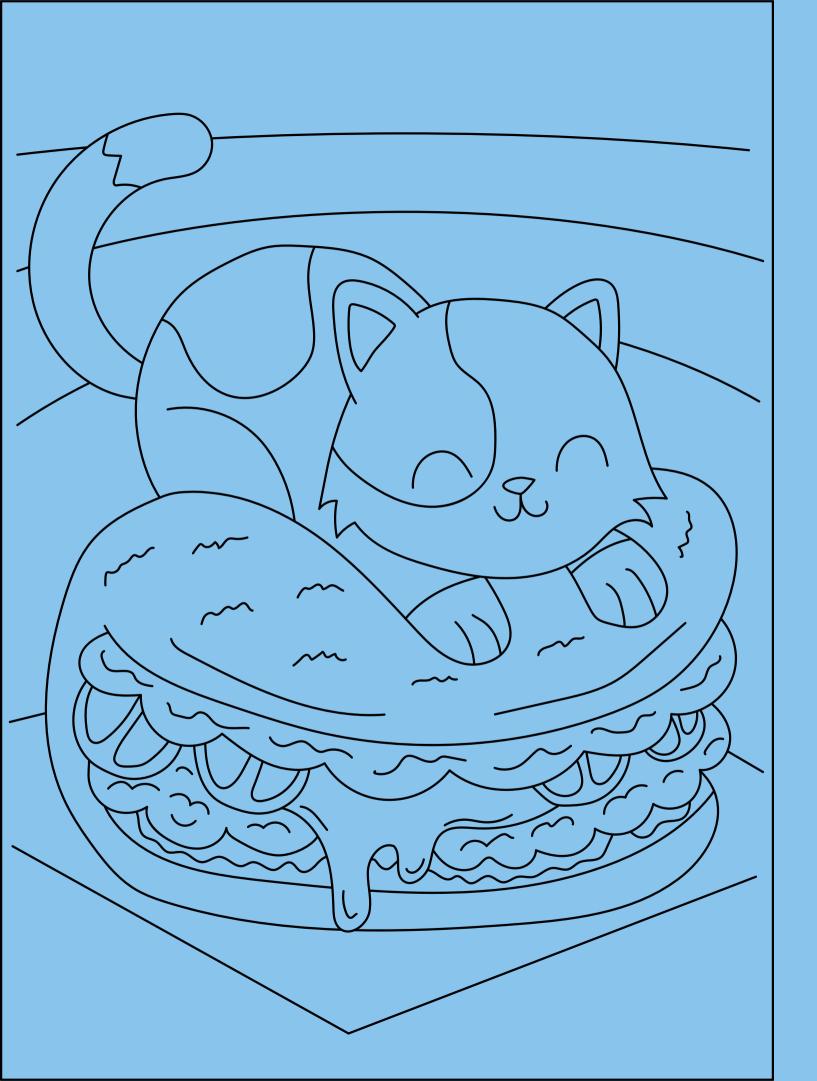
A certain Russian mathematician's decision regarding the first in a list of 7 was based on his belief that his contribution was lower than Richard Hamilton's, and his "disagreement with the organized mathematical community". Who am I talking about?



#### Compute

$$\int\limits_{0}^{\infty} rac{sin(x^3)}{x} dx$$





PHEW!
Greatheee



#### SOLUTIONS

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[1, 2]

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#### Paul Erdős

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$$\frac{1}{n+1}$$

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$$xy(x^2-y^2) + yz(y^2-z^2) + zx(z^2-x^2) = 1$$

# No such triplets exist

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(3, 2)

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#### Both are equal

Find the area of the surface  $A \cap B$ , where

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(2, 3)

Determine the value of

$$\int\limits_{0}^{20}(x^{2}-\lfloor x \rfloor \lceil x \rceil)dx$$

Determine the value of

$$\int\limits_0^{20}(x^2-\lfloor x\rfloor\lceil x
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$$\frac{20}{3}$$

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Grigori Perelman; he solved the Poincaré conjecture, one of the Millenium Prize problems, but refused to take the one million dollar prize.

Compute

$$\int\limits_{0}^{\infty} rac{sin(x^3)}{x} dx$$

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<u>6</u>