

1V1

POWERUP  
ROUND

# Rules

## Match-ups:

Every team faces every other team one at a time in a faceoff.

First team to answer amongst the teams facing off gets +30 for a correct answer and -15 for an incorrect answer. The other team gets -10 in case of a correct answer and +15 in case of an incorrect answer. If both don't answer within 2.5 minutes, both receive -10.

Rest of the teams can answer the question for +10/-5 marks before the teams facing off answer (or till the timer ends)

# Rules

Every team is given 2 powerups, one of each type.

These can be used only within the first 5 seconds of displaying the question and cannot be used again once used

High Stakes Powerup:- Using this powerup would make it high stakes for the other team. If you use this powerup the points for the other team become +50 for a correct answer, -20 for not attempting (irrespective of whether the other team gets correct or not) and -40 for an incorrect answer.

Sabotage Powerup:- Using this powerup you can prevent the other team from answering that question without a negative 10 penalty.



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# Team 1 vs Team 6

Let triangle ABC lie on the plane

$$x + y + z = 4$$

with centroid G. Suppose there exists a point  $P_1$  in space satisfying

$$P_1A^2 + P_1B^2 + P_1C^2 = 4P_1G^2$$

Let  $C'$  denote the curve formed by the intersection of the locus of all such points  $P$  with the plane of triangle ABC, and let  $d_1$  be the shortest distance from  $G$  to  $C'$ .

Let there exist another point  $P_2$  on the line joining  $G$  and  $H$  (the orthocenter of triangle ABC) such that,

$$d_1^2 + 3d_2^2 = 3R^2$$

where  $d_2$  is the distance from  $G$  to  $P_2$ , and  $R$  is the circumradius of triangle ABC. Determine the locus of point  $P_2$  (with the correct location, e.g. “line” is not sufficient, specify “line through X, Y”)



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Answer

Circle through  $O$  centered at  $G$



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# Team 2 vs Team 5

Nitanshu loves his girlfriend and wants to spend the rest of his life with her, but when he finally gathers up the courage to propose to her, she puts up a condition

She pulls 6 identical strands of string and holds them in her palm so their ends coincide. She randomly ties three knots on either end of various strings (a total of 6 knots)

She agrees to marry Nitanshu only if the strings form a loop when she pulls them apart. What is the probability that Nitanshu does not get to marry the love of his life?



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# Answer

7/15



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# Team 3 vs Team 4

Against all odds, Nitanshu finally gets married to the love of his life. Nitanshu now has two children. One of them is a girl, born on a Saturday. What is the probability that he has two girls?



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# Answer

13/27



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# Team 1 vs Team 5

Find the sum of areas of all distinct triangles whose vertices are selected from the set of vertices of a regular hexagon of area 1.



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# Answer

6



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# Team 2 vs Team 4

Before being happily married to the love of his life, Nitanshu's journey to true love was statistically impressive

He set out to have successful dates with 5 different types of girls. On any given evening, there is an equal chance of meeting any one of the 5 types.

But the probability of a successful date depends on his mood: in a great mood, his success probability is 0.6, and in a bad mood, it is 0.1. He started with a great mood and only fell in a bad mood if his previous date was unsuccessful

Nitanshu stopped only when he had a successful date with each of the 5 types at least once. How many dates do you think Nitanshu went on?



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# Answer

57



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# Team 3 vs Team 6

For distinct reals  $a, b, c$ . Find the largest  $m$  such that the following expression is always greater than  $m$ :

$$\left(\frac{a}{b-c}\right)^2 + \left(\frac{b}{c-a}\right)^2 + \left(\frac{c}{a-b}\right)^2$$



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# Answer

2



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# Team 1 vs Team 4

Let ABCD be a square. Points K and L are chosen randomly on the sides AB and AD respectively such that  $AK = AL$ .

Let P be the intersection point of the segments CL and DK. Let  $x$  be the smaller angle (there will be two angles, take the smaller one) between CL and DK. As K varies along the segment AB (excluding the endpoints A and B), the value of  $x$  varies. Let  $M$  be the maximum possible value of  $x$  and  $m$  be the minimum possible value of  $x$ .

$$\text{Find } \left(\frac{M}{m}\right)^2$$



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# Answer

4



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# Team 2 vs Team 6

Find the value of the following limit

$$\lim_{n \rightarrow \infty} \sum_{k=0}^n \frac{\binom{n}{k}}{n^k (k+3)}$$



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Answer

e - 2



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# Team 3 vs Team 5

Determine all pairs of complex numbers  $a, b$   
satisfying

$$|a| = |b| = 1, \quad \text{and} \quad a + b + a\bar{b} \in \mathbb{R}$$



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# Answer

$$a = 1, b = -1, a = -b$$



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# Team 4 vs Team 6

The length of the curve

$$x + y = \lfloor x^2 + y^2 \rfloor$$

is

$$a + \sqrt{b} - \sqrt{c}$$

where  $a, b, c$  are non-negative integers. Find

$$a \times b \times c$$



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# Answer

24



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# Team 1 vs Team 3

Find the value of the following limit

$$\lim_{n \rightarrow \infty} \frac{1}{n^2} \sum_{i=0}^n \sqrt{\binom{n+i}{2}}$$



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# Answer

$$\frac{3}{2\sqrt{2}}$$



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# Team 4 vs Team 5

$f$  is a quadratic polynomial such that for any 2 distinct real numbers  $x, y$

$$f(x) = f(y) \implies f(x^2 - 6y - 1) = f(y^2 + 8)$$

Find the value of

$$\frac{f(6) - f(2)}{f(3) - f(1)}$$



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# Answer

$$336/96 = 7/2 = 3.5$$



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# Team 1 vs Team 2

Pulkit is tasked with guarding pizza boxes for an event, he secretly steals 11 of them, each with an independent 50% chance of being 'Normal' or 'Double Cheese'. If Pulkit arranges them in a circle, what's the probability that there is at least one pair of adjacent 'Double Cheese' pizzas?



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Answer

1849/2048



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# Team 6 vs Team 5

Darsh wants to create a fair game for his friends, Shivansh and Aryan, by dropping a square hoop of side unity on a floor ruled with equidistant parallel lines. Shivansh bets the hoop will cross a line, while Aryan bets it won't. To make the game fair, what distance ' $d$ ' must Darsh set between the parallel lines?



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# Answer

$$\frac{8}{\pi}$$



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# Team 2 vs Team 3

Pulkit and Shivansh play a game. Pulkit goes first and can choose any natural number and add it to the current sum which is initially 0. Then Shivansh chooses a natural number amongst  $x$  or  $x - 1$  where  $x$  is the number chosen by Pulkit and adds it to the sum.

This process continues. When the sum reaches exactly 10, the last player to select the number wins, if the sum exceeds 10 then he loses. List all possible initial choices of Pulkit so that he can win the game



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# Answer

2, 3, 6, 7, 8, 9, 10



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# Team 1 vs Team 4 (Redo)

For each continuous function  $f: [0, 1] \rightarrow \mathbb{R}$  let

$$I(f) = \int_0^1 (x^2 f(x) - x(f(x))^2) dx$$

Find the maximum value of  $I(f)$  over all functions  $f$



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Answer

$1/16$



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# Audience

The largest known prime is  $2^{136279841} - 1$ , This number is a type of prime called a \_\_\_\_\_ prime and is the \_\_\_\_th prime of this type



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Answer

Mersenne, 52



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Audience

Name all Millenium Prize Problems



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Answer

P vs NP Problem

Navier–Stokes Existence and Smoothness

Riemann Hypothesis

Yang–Mills Existence and Mass Gap

Birch and Swinnerton-Dyer Conjecture

Hodge Conjecture

Poincaré Conjecture